

# ETF Casestudy

Peter Kempthorne

2024-09-23

## Contents

<b>0. Load Libraries</b>	<b>2</b>
<b>1. Load data into R session</b>	<b>4</b>
1.1 Daily Stock Price Data from Yahoo . . . . .	4
1.2 Weekly Stock Price Data . . . . .	6
1.3 Save data objects for casestudy in R data file . . . . .	9
<b>2. Analysis of returns for specific period (2013-2024)</b>	<b>10</b>
2.1 Subset data objects for specific period . . . . .	10
2.2 Define y0 equal to weekly returns of specific sector etf (XLP) . . . . .	10
2.3 Define df0 equal to data frame with y0 and index etfs . . . . .	11
2.4 Print out summary statistics for weekly returns . . . . .	11
2.5 Compute/plot cumulative returns . . . . .	12
<b>3. Regression of Sector ETF on Index ETFs</b>	<b>14</b>
3.1 Fit regression of y0 on index etf returns . . . . .	14
3.2 Plot Actual versus Fitted Values . . . . .	14
<b>4. Use R package car to analyse residuals/influence</b>	<b>16</b>
4.1 plot() method gives 4-panel plots of residuals and Leverage . . . . .	16
4.3 influencePlot() output . . . . .	17
4.4 residualPlots() . . . . .	18
<b>5. Compare Regression Using Z-scores of Predictors</b>	<b>20</b>
5.1 Reproduce Linear Regression on Predictors . . . . .	20
5.2 Repeat regression with Z-scores of index etfs . . . . .	21
<b>6. PCA of Explanatory Variables</b>	<b>22</b>
6.1 Extract scaled x matrix (except intercept) . . . . .	22
6.2 Compute PCA of x . . . . .	22
6.3 The Screeplot: Barplot of Principal Component Variances . . . . .	22
<b>7. Regressions on PC variables</b>	<b>26</b>
7.1 Univariate regressions . . . . .	26
7.2 Regression model on all PC variables . . . . .	30
7.3 Use PCA regression to recompute regression parameters . . . . .	31
7.4 Compute regression parameter based on only first 3 pc vars . . . . .	31
7.5 Compute regression parameter based on significant pc vars . . . . .	31
7.6 Create table of betas from these three fits . . . . .	31
7.7 Demonstrate equality of LS and PCA Regression Fitted Values . . . . .	32

<b>8. Ridge and Lasso Regression Fits</b>	<b>34</b>
8.1 Load glmnet package for ridge/lasso regressions . . . . .	34
8.2 Define y vector and x matrix for ridge/lasso fits . . . . .	34
8.3 Plot coefficient trace of ridge regression model . . . . .	34
8.4 Plot coefficient trace of lasso regression model — . . . . .	34
8.5 Apply Cross Validation to choose ridge parameters . . . . .	35
	<b>39</b>
8.6 Apply Cross Validation to choose lasso parameters . . . . .	39

## 0. Load Libraries

```
library("quantmod")
```

```
## Loading required package: xts
```

```
## Loading required package: zoo
```

```
##
```

```
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
```

```
##
```

```
##      as.Date, as.Date.numeric
```

```
## Loading required package: TTR
```

```
## Registered S3 method overwritten by 'quantmod':
```

```
##      method          from
```

```
##      as.zoo.data.frame zoo
```

```
library("tseries")
```

```
library("ggplot2")
```

```
library("reshape2")
```

```
library("tidyverse")
```

```
## -- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
```

```
## v dplyr      1.1.4      v readr      2.1.5
```

```
## v forcats    1.0.0      v stringr    1.5.1
```

```
## v lubridate  1.9.3      v tibble     3.2.1
```

```
## v purrr      1.0.2      v tidyr      1.3.1
```

```
## -- Conflicts ----- tidyverse_conflicts() --
```

```
## x dplyr::filter() masks stats::filter()
```

```
## x dplyr::first()  masks xts::first()
```

```
## x dplyr::lag()    masks stats::lag()
```

```
## x dplyr::last()   masks xts::last()
```

```
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library("tidyquant")
```

```
## -- Attaching core tidyquant packages ----- tidyquant 1.0.9 --
```

```
## v PerformanceAnalytics 2.0.4
```

```
## -- Conflicts ----- tidyquant_conflicts() --
```

```
## x zoo::as.Date()          masks base::as.Date()
```

```
## x zoo::as.Date.numeric()  masks base::as.Date.numeric()
```

```
## x dplyr::filter()         masks stats::filter()
```

```
## x dplyr::first()          masks xts::first()
```

```

## x dplyr::lag()           masks stats::lag()
## x dplyr::last()          masks xts::last()
## x PerformanceAnalytics::legend() masks graphics::legend()
## x quantmod::summary()     masks base::summary()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
library("car")

## Loading required package: carData
##
## Attaching package: 'car'
##
## The following object is masked from 'package:dplyr':
##
##     recode
##
## The following object is masked from 'package:purrr':
##
##     some
library("ggfortify")
library("glmnet")

## Loading required package: Matrix
##
## Attaching package: 'Matrix'
##
## The following objects are masked from 'package:tidyr':
##
##     expand, pack, unpack
##
## Loaded glmnet 4.1-8

```

# 1. Load data into R session

## 1.1 Daily Stock Price Data from Yahoo

Apply quantmod(sub-package TTR) function: getYahoodata

Returns historical data for any symbol at the website

<http://finance.yahoo.com>

### 1.1.1 Set start and end date for collection in YYYYMMDD (numeric) format

```
date.start<-"2000-01-01"  
date.end<-"2024-09-20"
```

### 1.1.2 Input list of ETF symbols

```
options(stringsAsFactors=FALSE)  
list.etf0.labels0<-read.table(file="spymod_symbols_labels.csv",sep=",", header=FALSE)  
list.etf0<-as.character(list.etf0.labels0[,1])
```

```
for (etf0 in list.etf0){  
  #etf0<-"XLB"  
  getSymbols(etf0, from=date.start, to=date.end)  
  print(etf0)  
}
```

```
## [1] "SPY"  
## [1] "MDY"  
## [1] "QQQ"  
## [1] "XLB"  
## [1] "XLV"  
## [1] "XLP"  
## [1] "XLY"  
## [1] "XLE"  
## [1] "XLF"  
## [1] "XLI"  
## [1] "XLK"  
## [1] "XLU"  
## [1] "DIA"
```

### 1.1.3 Construct matrix of closing prices

```
etf0.pmat0<-matrix(NA, nrow=NROW(SPY),ncol=length(list.etf0))  
dimnames(etf0.pmat0)<-list(dimnames(SPY)[[1]], list.etf0)  
# Create zoo objects of just Adjusted prices  
for (j.etf0 in c(1:length(list.etf0))){  
  #j.etf0<-3  
  j.etf0.name0<-list.etf0[j.etf0]  
  obj0.0<-get(list.etf0[j.etf0])[,6]  
  dimnames(obj0.0)[[2]]<-j.etf0.name0  
  obj0.0.name0<-paste(j.etf0.name0, ".0", sep="")  
  print(obj0.0.name0)
```

```
assign(obj0.0.name0, obj0.0)
}
```

```
## [1] "SPY.0"
## [1] "MDY.0"
## [1] "QQQ.0"
## [1] "XLB.0"
## [1] "XLV.0"
## [1] "XLP.0"
## [1] "XLY.0"
## [1] "XLE.0"
## [1] "XLF.0"
## [1] "XLI.0"
## [1] "XLK.0"
## [1] "XLU.0"
## [1] "DIA.0"
```

```
list.etf0.0<-paste(list.etf0, ".0", sep="")
```

```
pmat0.0<-eval(parse(text=paste("merge(",
                                paste(list.etf0.0, collapse=", "),
                                ")", collapse="")))
                                ")", collapse=""))
```

#### 1.1.4 Plot time series of daily SPY prices

```
plot(pmat0.0[, "SPY"])
```



```
chartSeries(pmat0.0[, "XLF"])
```



## 1.2 Weekly Stock Price Data

### 1.2.1 Create matrix of weekly closing prices

```
pmat0.0.1.weekly<-to.weekly(pmat0.0[,1])
pmat0.0.weekly.coredata<-matrix(NA, nrow=nrow(pmat0.0.1.weekly), ncol=ncol(pmat0.0))

for (jcol0 in c(1:ncol(pmat0.0))) {
  pmat0.0.weekly.coredata[,jcol0]<- to.weekly(pmat0.0[,jcol0])[,4]
}

dim(pmat0.0.weekly.coredata)

## [1] 1290 13

length(time(pmat0.0.1.weekly))

## [1] 1290

as.numeric(time(pmat0.0.1.weekly)[1:5])

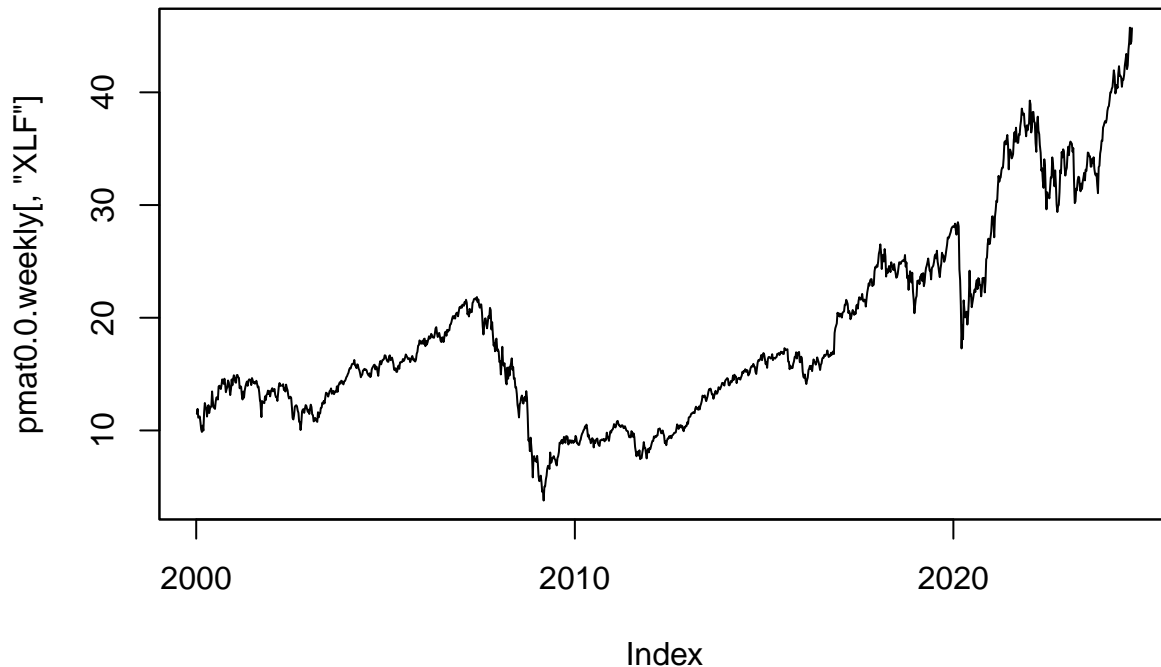
## [1] 10963 10970 10977 10984 10991

pmat0.0.weekly<-zoo(pmat0.0.weekly.coredata, order.by=time(pmat0.0.1.weekly))
```

```
dimnames(pmat0.0.weekly)[[2]]<-dimnames(pmat0.0)[[2]]
```

### 1.2.2 Create matrix of weekly log returns

```
plot(pmat0.0.weekly[, "XLF"])
```



```
head(pmat0.0.weekly)
```

```
##          SPY      MDY      QQQ      XLB      XLV      XLP      XLY
## 2000-01-07 93.20474 60.77375 76.63248 15.80843 20.71107 13.61024 22.95816
## 2000-01-14 93.98414 61.94609 79.50619 15.29558 22.30757 13.48810 22.64126
## 2000-01-21 92.36544 62.27434 81.95415 14.31486 22.19897 13.02570 21.70226
## 2000-01-28 86.88984 59.03870 73.22663 13.51409 21.02603 12.73780 20.28206
## 2000-02-04 91.18630 61.12546 82.72588 13.48710 21.99261 12.68545 21.08020
## 2000-02-11 88.68842 61.19578 85.04080 12.50639 21.30840 12.04856 20.27033
##          XLE      XLF      XLI      XLK      XLU      DIA
## 2000-01-07 14.88110 11.47026 18.86840 38.36602 11.60620 67.18431
## 2000-01-14 14.87281 11.91383 18.62753 39.77654 11.38846 68.34761
## 2000-01-21 15.41936 11.12786 18.19596 40.34076 11.77115 65.66428
## 2000-01-28 14.14408 11.17456 17.34287 36.81446 11.15752 62.51753
## 2000-02-04 13.94533 11.24459 17.41313 40.71688 11.44784 63.88174
## 2000-02-11 13.63065 10.70765 16.35930 40.90496 11.07835 60.82587
```

```
rmat0.0.weekly<-na.omit(diff(log(pmat0.0.weekly)))
```

Table frequency of return horizon in days. Note that September 11, 2001 corresponds to 11.

```
table(diff(time(rmat0.0.weekly)))
```

```
##
##    3    6    7    8   11
##    1   39 1209   38    1
```

```
cor(rmat0.0.weekly)
```

```
##          SPY          MDY          QQQ          XLB          XLV          XLP          XLY
## SPY 1.0000000 0.9263244 0.8406530 0.8169412 0.8048856 0.7133526 0.8851434
## MDY 0.9263244 1.0000000 0.7857591 0.8350609 0.7038070 0.6111273 0.8456266
## QQQ 0.8406530 0.7857591 1.0000000 0.5875579 0.6467435 0.4414167 0.7333358
## XLB 0.8169412 0.8350609 0.5875579 1.0000000 0.6174494 0.5939057 0.7566491
## XLV 0.8048856 0.7038070 0.6467435 0.6174494 1.0000000 0.6477534 0.6718096
## XLP 0.7133526 0.6111273 0.4414167 0.5939057 0.6477534 1.0000000 0.6295337
## XLY 0.8851434 0.8456266 0.7333358 0.7566491 0.6718096 0.6295337 1.0000000
## XLE 0.6713572 0.7040841 0.4252876 0.7034908 0.4825468 0.4657262 0.5259733
## XLF 0.8404444 0.8232555 0.5853957 0.7296194 0.6391777 0.5685931 0.7733551
## XLI 0.9091362 0.9002693 0.7081750 0.8512075 0.7054340 0.6662999 0.8287766
## XLK 0.8542522 0.7790903 0.9640273 0.6072689 0.6348147 0.4588322 0.7419041
## XLU 0.6166184 0.5736058 0.3873537 0.5073337 0.5512258 0.6180840 0.4970907
## DIA 0.9556731 0.8819201 0.7305154 0.8294260 0.7928100 0.7558884 0.8572597
##          XLE          XLF          XLI          XLK          XLU          DIA
## SPY 0.6713572 0.8404444 0.9091362 0.8542522 0.6166184 0.9556731
## MDY 0.7040841 0.8232555 0.9002693 0.7790903 0.5736058 0.8819201
## QQQ 0.4252876 0.5853957 0.7081750 0.9640273 0.3873537 0.7305154
## XLB 0.7034908 0.7296194 0.8512075 0.6072689 0.5073337 0.8294260
## XLV 0.4825468 0.6391777 0.7054340 0.6348147 0.5512258 0.7928100
## XLP 0.4657262 0.5685931 0.6662999 0.4588322 0.6180840 0.7558884
## XLY 0.5259733 0.7733551 0.8287766 0.7419041 0.4970907 0.8572597
## XLE 1.0000000 0.5858364 0.6812261 0.4344610 0.4907898 0.6729492
## XLF 0.5858364 1.0000000 0.8241749 0.5965313 0.4832339 0.8299180
## XLI 0.6812261 0.8241749 1.0000000 0.7224326 0.5653708 0.9232702
## XLK 0.4344610 0.5965313 0.7224326 1.0000000 0.4106982 0.7606385
## XLU 0.4907898 0.4832339 0.5653708 0.4106982 1.0000000 0.6176370
## DIA 0.6729492 0.8299180 0.9232702 0.7606385 0.6176370 1.0000000
```

```
dim(rmat0.0.weekly)
```

```
## [1] 1289    13
```

### 1.2.3 Create separate objects for sector etfs and index etfs

```
rmat0.0.weekly.sectoretfs<-rmat0.0.weekly[,c(4:12)]
list.sectoretfs<-dimnames(rmat0.0.weekly.sectoretfs)[[2]]
list.sectoretfs.labels<-list.etf0.labels0[4:12,2]
list.sectoretfs.labels.0<-paste(list.sectoretfs.labels,"(",list.sectoretfs,")",sep="")

rmat0.0.weekly.indexetfs<-rmat0.0.weekly[,c(1:3, 13)]

list.indexetfs<-dimnames(rmat0.0.weekly.indexetfs)[[2]]

head(rmat0.0.weekly.indexetfs)
```

```
##          SPY          MDY          QQQ          DIA
## 2000-01-14 0.008327424 0.019106545 0.03681378 0.01716685
## 2000-01-21 -0.017373142 0.005284925 0.03032506 -0.04005145
## 2000-01-28 -0.061111787 -0.053356329 -0.11260074 -0.04910812
## 2000-02-04 0.048263591 0.034735339 0.12197327 0.02158654
## 2000-02-11 -0.027775413 0.001149762 0.02759872 -0.04901837
```



```
## 2000-02-18 -0.024636530 -0.013889528 -0.01449766 -0.02053414
list.sectoretfs<-dimnames(rmat0.0.weekly.sectoretfs)[[2]]
list.sectoretfs.labels<-list.etf0.labels0[4:12,2]
list.sectoretfs.labels.0<-paste(list.sectoretfs.labels,"(",list.sectoretfs,")",sep="")

rmat0.0.weekly.indexetfs<-rmat0.0.weekly[,c(1:3, 13)]

list.indexetfs<-dimnames(rmat0.0.weekly.indexetfs)[[2]]
```

### 1.3 Save data objects for casestudy in R data file

```
list.obj.tosave0<-c(
  "pmat0.0.weekly",
  "rmat0.0.weekly.sectoretfs",
  "rmat0.0.weekly.indexetfs",
  "list.sectoretfs",
  "list.sectoretfs.labels",
  "list.sectoretfs.labels.0",
  "list.indexetfs")

save(file="casestudy_1_0_etfs.RData", list=list.obj.tosave0)
```

## 2. Analysis of returns for specific period (2013-2024)

### 2.1 Subset data objects for specific period

```
period0.startyear0<-"2013"
period0.endyear0<-"2024"
period0.startdate0<-as.Date(paste(period0.startyear0,"-01-01",sep=""))
period0.enddate0<-as.Date(paste(period0.endyear0,"-12-31",sep=""))
period0.label0<-paste("Period: ", period0.startyear0, "-" , period0.endyear0,sep="")

rmat0.0.weekly.sectoretfs.period0<-window(rmat0.0.weekly.sectoretfs,
                                          start = period0.startdate0,
                                          end=period0.enddate0)

rmat0.0.weekly.indexetfs.period0<-window(rmat0.0.weekly.indexetfs,
                                          start = period0.startdate0,
                                          end=period0.enddate0)

tail(rmat0.0.weekly.sectoretfs.period0)
```

##		XLB	XLV	XLP	XLV	XLE
##	2024-08-16	0.02244277	0.019085932	0.017036621	0.049188915	0.0117857438
##	2024-08-23	0.02356905	0.016736262	0.016384650	0.025553898	-0.0008846825
##	2024-08-30	0.01657698	0.011194744	0.008153447	-0.004000112	0.0097980571
##	2024-09-06	-0.04768237	-0.020891006	0.005800569	-0.025548052	-0.0594680988
##	2024-09-13	0.03067392	0.014253292	0.011263029	0.053790186	-0.0048951823
##	2024-09-19	0.02187566	-0.001538103	-0.015368279	0.023921903	0.0386184091
##		XLF	XLI	XLK	XLU	
##	2024-08-16	0.031800263	0.02141377	0.07389253	0.011212570	
##	2024-08-23	0.014966262	0.01802974	0.01149650	0.013079003	
##	2024-08-30	0.029058301	0.01672209	-0.01634106	0.011469413	
##	2024-09-06	-0.032214287	-0.04329417	-0.07739868	-0.004993404	
##	2024-09-13	0.004954907	0.03650103	0.07789781	0.034059585	
##	2024-09-19	0.026165580	0.02682535	0.01432260	-0.006642900	

```
tail(rmat0.0.weekly.indexetfs.period0)
```

##		SPY	MDY	QQQ	DIA
##	2024-08-16	0.039221464	0.026451262	0.053219670	0.02971464
##	2024-08-23	0.014009032	0.028103817	0.010408147	0.01243647
##	2024-08-30	0.002753516	-0.001537826	-0.007801207	0.01057932
##	2024-09-06	-0.042251072	-0.049590953	-0.059652666	-0.02861231
##	2024-09-13	0.039284067	0.031976940	0.057698098	0.02555637
##	2024-09-19	0.015834531	0.029501289	0.016731358	0.01547228

### 2.2 Define y0 equal to weekly returns of specific sector etf (XLP)

```
y0.name="XLP (Consumer Staples ETF)"
y0.symbol="XLP"
y0<-rmat0.0.weekly.sectoretfs.period0$XLP
```

The following R code can be copied above to change the choice of sector etf

```
if (FALSE) {
# 1.1 Change to XLK Technology:
```

```

# Define y0 equal to weekly returns of sector etf XLK
y0.name="XLK (Technology ETF)"
y0.symbol="XLK"
y0<-rmat0.0.weekly.sectoretfs.period0$XLK

# 1.2 Change to XLI Industrials:
# Define y0 equal to weekly returns of sector etf XLI
y0.name="XLI (Industrials ETF)"
y0.symbol="XLI"
y0<-rmat0.0.weekly.sectoretfs.period0$XLI

}

```

## 2.3 Define df0 equal to data frame with y0 and index etfs

```
df0<-data.frame(cbind(y0,rmat0.0.weekly.indexetfs.period0))
```

## 2.4 Print out summary statistics for weekly returns

### 2.4.1 Annual means, volatilities

```
apply(df0,2,summary)
```

```

##              y0              SPY              MDY              QQQ              DIA
## Min.      -0.116194529 -0.157189170 -0.202701342 -0.119356994 -0.187241821
## 1st Qu.   -0.007267913 -0.007898118 -0.010483530 -0.010700447 -0.007719788
## Median    0.003452617  0.004176716  0.003299196  0.004865223  0.003425232
## Mean      0.001926583  0.002634949  0.002100222  0.003474893  0.002332767
## 3rd Qu.   0.012286899  0.014691387  0.015611677  0.019312326  0.013328769
## Max.      0.065588245  0.114145893  0.172891253  0.091102251  0.119922619

```

```

mean.annual<-apply(df0,2,mean)*52
vol.annual<-sqrt(52*apply(df0,2,var))
data.frame(mean=mean.annual, vol=vol.annual)

```

```

##          mean          vol
## y0  0.1001823 0.1370785
## SPY 0.1370173 0.1607707
## MDY 0.1092116 0.1957695
## QQQ 0.1806944 0.1893826
## DIA 0.1213039 0.1628826

```

### 2.4.2 Pairwise correlations and pairs plots

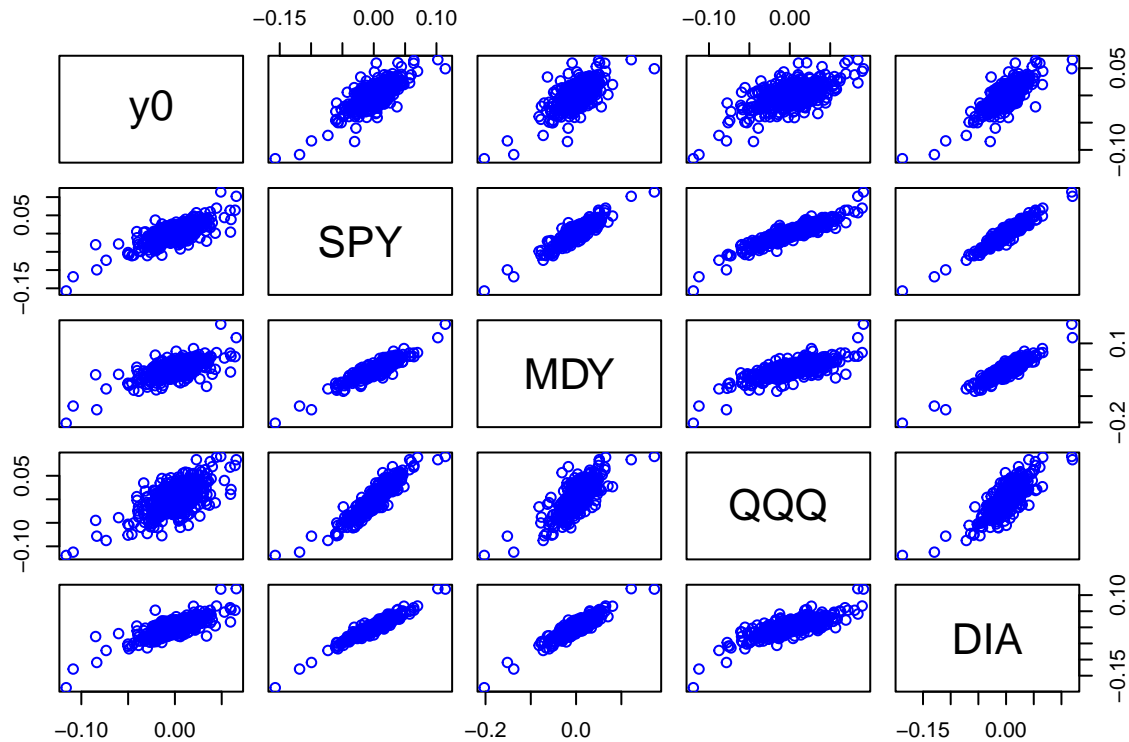
```
cor(df0)
```

```

##              y0              SPY              MDY              QQQ              DIA
## y0  1.0000000 0.7526003 0.6552977 0.6243527 0.7768283
## SPY 0.7526003 1.0000000 0.9175267 0.9181598 0.9540900
## MDY 0.6552977 0.9175267 1.0000000 0.7689490 0.9092160
## QQQ 0.6243527 0.9181598 0.7689490 1.0000000 0.7977508
## DIA 0.7768283 0.9540900 0.9092160 0.7977508 1.0000000

```

```
pairs(df0,col='blue')
```



## 2.5 Compute/plot cumulative returns

```
# Convert df0 to cumulative return series  
# All starting at 1.
```

```
df0cumret<-data.frame(exp(apply(log(1+df0),2,cumsum)))  
names(df0cumret)
```

```
## [1] "y0" "SPY" "MDY" "QQQ" "DIA"
```

```
names(df0cumret)[1]<-y0.symbol
```

```
# Rename y0 to actual symbol for plotting
```

```
# Add Date variable (after computing cumulative returns)
```

```
df0cumret$Date<-time(rmat0.0.weekly.indexetfs.period0)
```

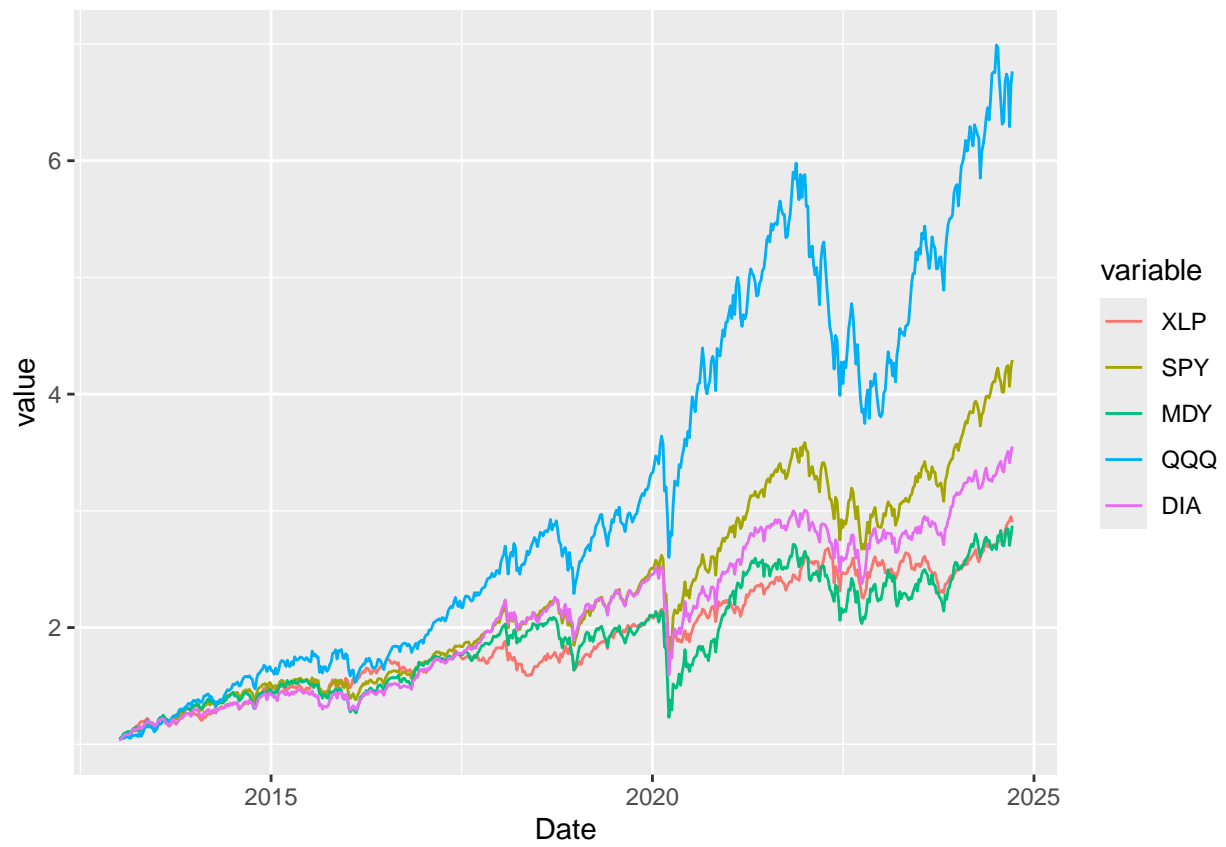
```
# Apply melt function from reshape2 package for using ggplot
```

```
meltdf0cumret <- melt(df0cumret,id="Date")
```

```
#names(meltdf)
```

```
#meltdf$variable
```

```
gg0<-ggplot(meltdf0cumret,aes(x=Date,y=value,colour=variable,group=variable)) +  
  geom_line()  
print(gg0)
```



### 3. Regression of Sector ETF on Index ETFs

#### 3.1 Fit regression of y0 on index etf returns

```
fit<-lm(y0 ~., data=df0); fit.summary<-summary(fit)

fit.summary

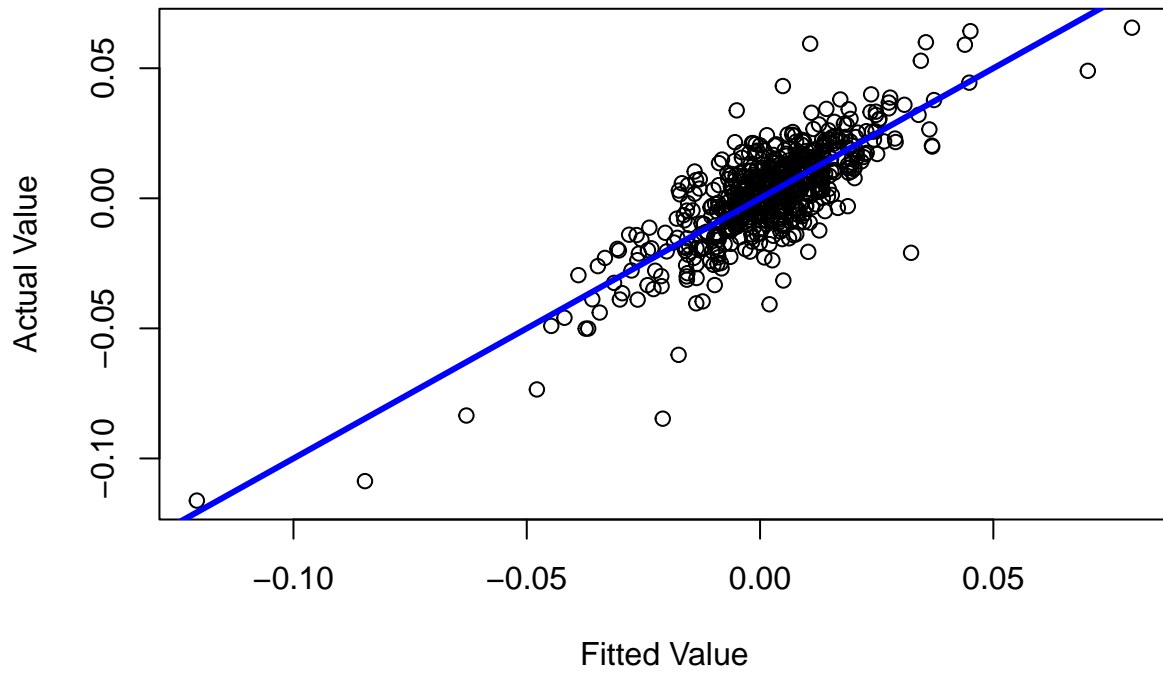
##
## Call:
## lm(formula = y0 ~ ., data = df0)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.063859 -0.006274  0.000033  0.007041  0.048673
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0002070  0.0004662   0.444   0.657
## SPY          1.0069903  0.1632673   6.168 1.27e-09 ***
## MDY         -0.3683498  0.0483853  -7.613 1.03e-13 ***
## QQQ         -0.3142920  0.0640313  -4.908 1.18e-06 ***
## DIA          0.3995081  0.0908524   4.397 1.29e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01141 on 607 degrees of freedom
## Multiple R-squared:  0.6418, Adjusted R-squared:  0.6394
## F-statistic: 271.9 on 4 and 607 DF,  p-value: < 2.2e-16
# Note list components of output from summary()
names(fit.summary)

## [1] "call"          "terms"          "residuals"      "coefficients"
## [5] "aliases"       "sigma"          "df"             "r.squared"
## [9] "adj.r.squared" "fstatistic"     "cov.unscaled"
# print method for fit.summary is condensed
```

#### 3.2 Plot Actual versus Fitted Values

```
plot(fit$fitted.values, df0$y0,
     xlab="Fitted Value",
     ylab = "Actual Value",
     main=paste(c("Regression Model","\n", y0.name,
                  "\nR-Squared =",
                  as.character(round(fit.summary$r.squared,digits=2))),
           collapse=""),
     abline(a=0,b=1,col="blue",lwd=3))
```

**Regression Model**  
**XLP (Consumer Staples ETF)**  
**R-Squared =0.64**

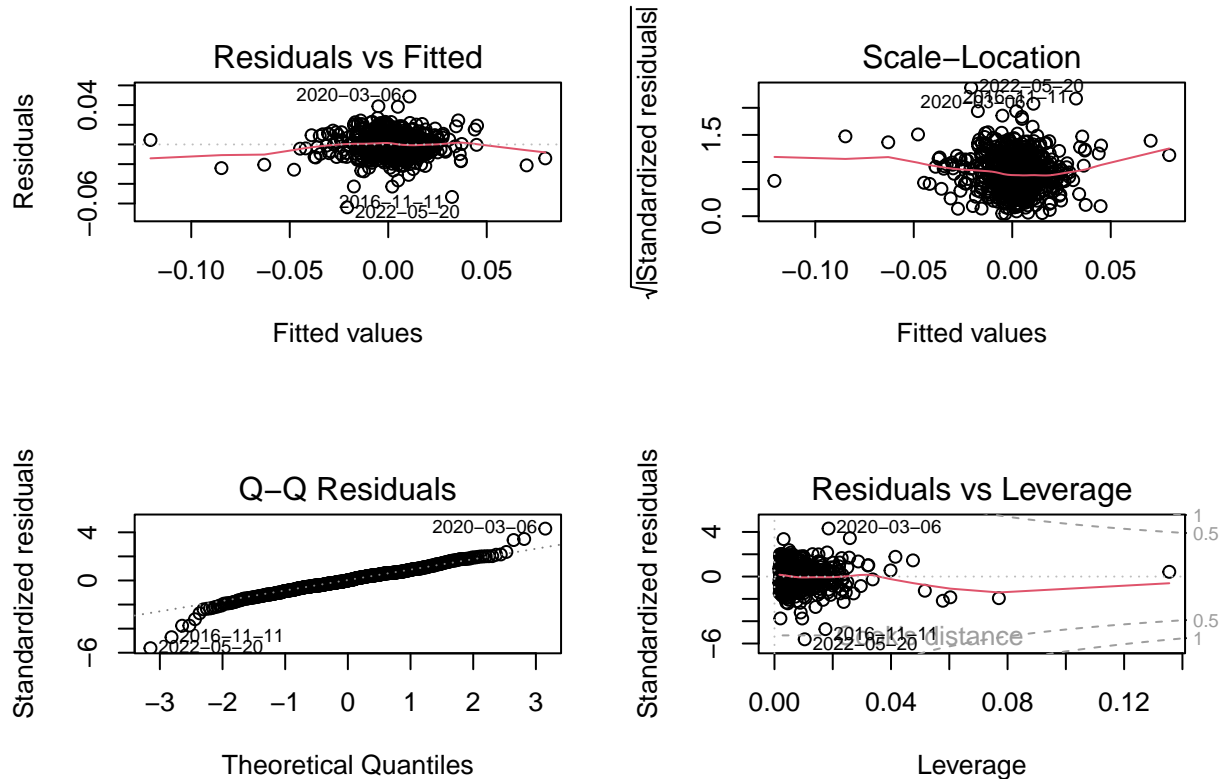


## 4. Use R package car to analyse residuals/influence

```
library(car)
```

### 4.1 plot() method gives 4-panel plots of residuals and Leverage

```
oldpar=par(no.readonly=TRUE)
layout(matrix(c(1,2,3,4),2,2)) # optional 4 graphs/page
plot(fit) ; par(oldpar,no.readonly=TRUE)
```

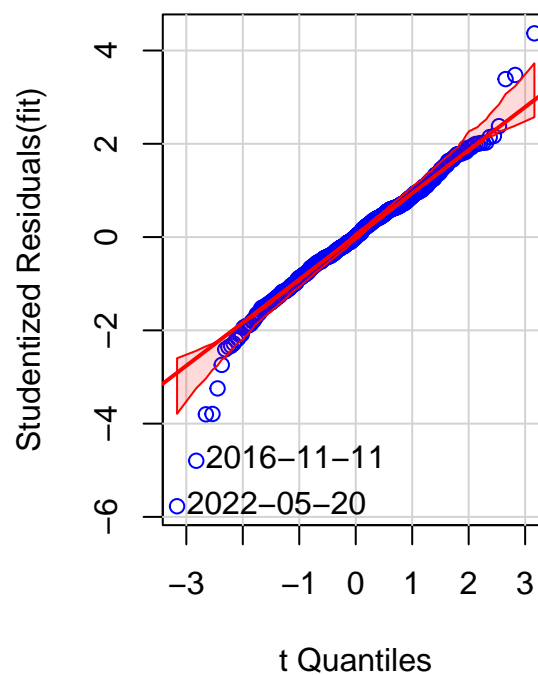
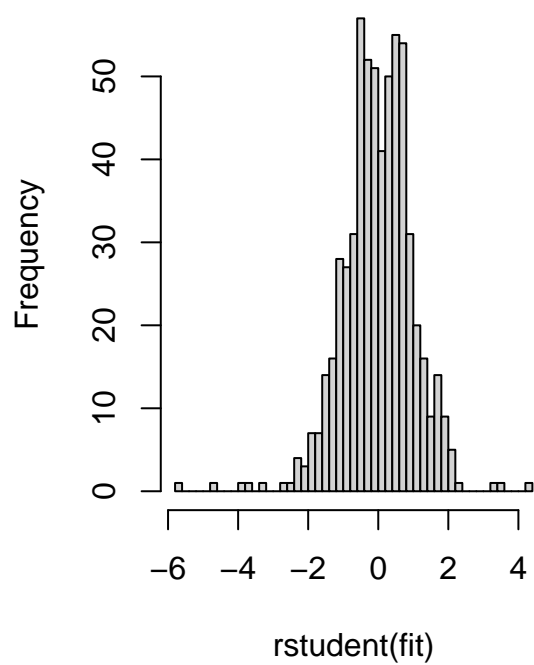


```
## 4.2 studres() Studentized residuals and qqPlot() -
```

```
# plot histogram and t-dist QQ plot
par(mfcol=c(1,2));
hist(rstudent(fit),breaks=50);
qqPlot(fit,col="blue",col.lines="red")
```



## Histogram of rstudent(fit)

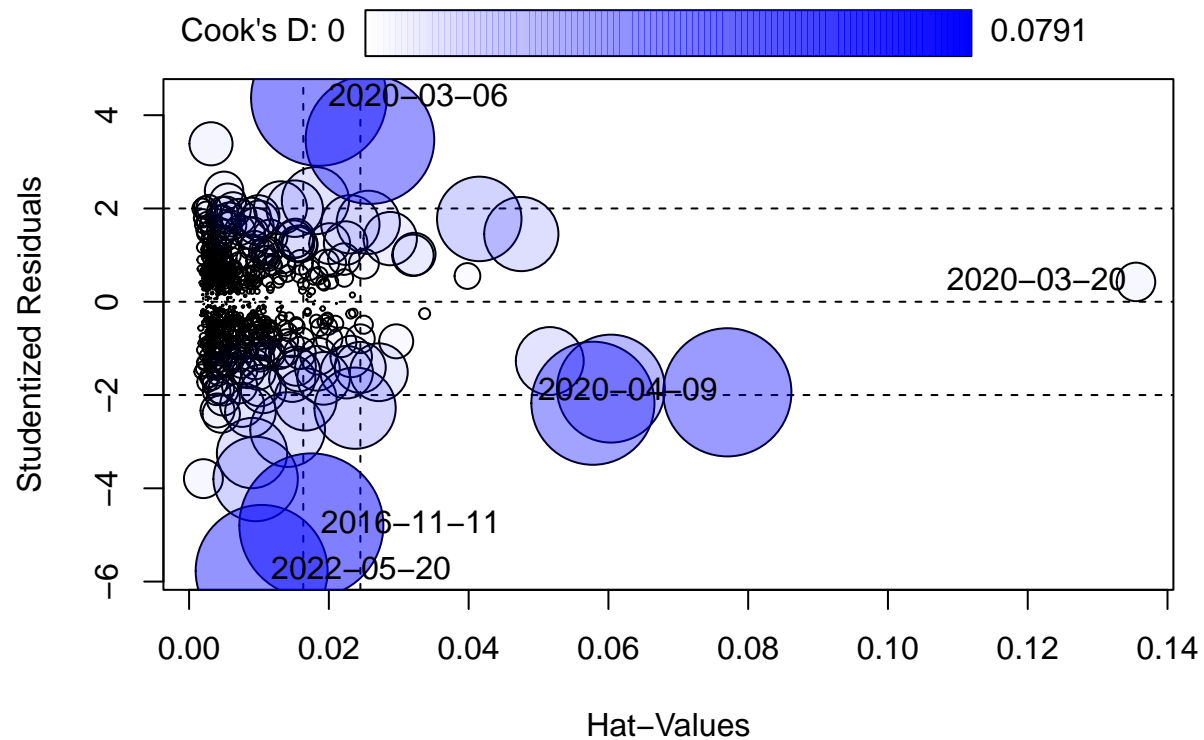


```
## 2016-11-11 2022-05-20
##          202          490
```

```
#
```

### 4.3 influencePlot() output

```
par(mfcol=c(1,1))
influencePlot(lm(y0~., data=df0))
```

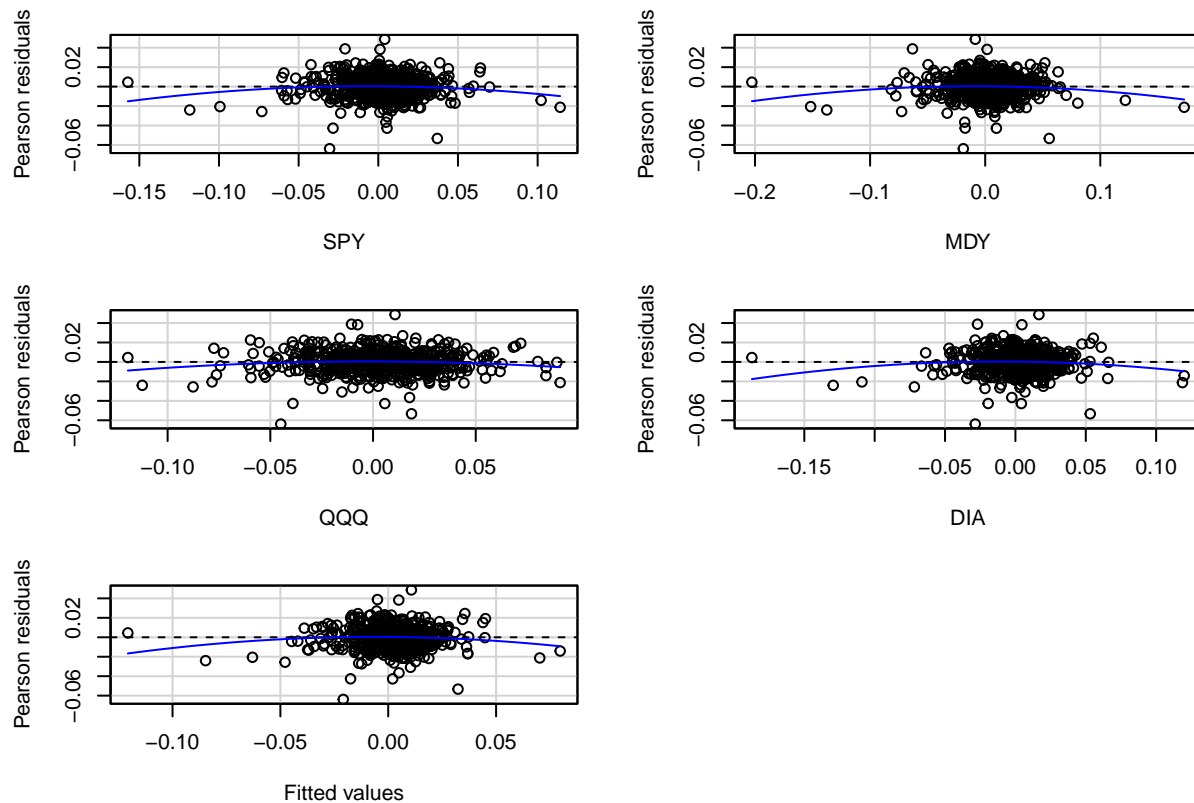


##		StudRes	Hat	CookD
##	2016-11-11	-4.7964026	0.01749576	0.079066753
##	2020-03-06	4.3678362	0.01857384	0.070123119
##	2020-03-20	0.4249459	0.13549138	0.005667955
##	2020-04-09	-1.9418203	0.07701466	0.062639579
##	2022-05-20	-5.7714057	0.01037971	0.066341866

#### 4.4 residualPlots()

Visually test for curvature in linear regression terms.

```
residualPlots(lm(y0~., data=df0), tests=FALSE)
```



Print out test results

```
# Test results (for curvature in plots)
residualPlots(lm(y0~., data=df0), tests=TRUE, plot=FALSE)

##           Test stat Pr(>|Test stat|)
## SPY          -2.2498      0.02482 *
## MDY          -2.2815      0.02287 *
## QQQ          -2.0270      0.04310 *
## DIA          -2.3455      0.01932 *
## Tukey test   -2.2004      0.02778 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## 5. Compare Regression Using Z-scores of Predictors

### 5.1 Reproduce Linear Regression on Predictors

```
# df0: data frame with security and indexes
# y0
# Regress y0 on indexes

# Add arguments to get x matrix and y
fit<-lm(y0 ~., data=df0,x=TRUE,y=TRUE);
fit.summary<-summary(fit)
fit.summary

##
## Call:
## lm(formula = y0 ~ ., data = df0, x = TRUE, y = TRUE)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.063859 -0.006274  0.000033  0.007041  0.048673
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0002070  0.0004662   0.444   0.657
## SPY          1.0069903  0.1632673   6.168 1.27e-09 ***
## MDY         -0.3683498  0.0483853  -7.613 1.03e-13 ***
## QQQ         -0.3142920  0.0640313  -4.908 1.18e-06 ***
## DIA          0.3995081  0.0908524   4.397 1.29e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01141 on 607 degrees of freedom
## Multiple R-squared:  0.6418, Adjusted R-squared:  0.6394
## F-statistic: 271.9 on 4 and 607 DF,  p-value: < 2.2e-16

fit$coefficients

##      (Intercept)          SPY          MDY          QQQ          DIA
## 0.0002070038  1.0069902798 -0.3683498406 -0.3142920210  0.3995081430

# Output argument x has matrix of predictor variables
head(fit$x)

##      (Intercept)          SPY          MDY          QQQ          DIA
## 2013-01-04      1 0.044280965 0.049746267 0.043715341 3.793252e-02
## 2013-01-11      1 0.004771257 0.001196815 0.009410744 4.910796e-03
## 2013-01-18      1 0.008530791 0.015020782 -0.002828828 1.118619e-02
## 2013-01-25      1 0.012861058 0.022542269 -0.001044416 1.841951e-02
## 2013-02-01      1 0.006567324 0.004198834 0.009802910 8.047782e-03
## 2013-02-08      1 0.003695823 0.007057868 0.004865621 -7.152686e-05
#
```

## 5.2 Repeat regression with Z-scores of index etfs

```
# The R function scale outputs z scores of data vectors
df0.zscore<-data.frame(cbind(y0,apply(rmat0.0.weekly.indexetfs.period0,2,scale)))
fit<-lm(y0 ~., data=df0.zscore, x=TRUE,y=TRUE); fit.summary<-summary(fit)
fit.summary
```

```
##
## Call:
## lm(formula = y0 ~ ., data = df0.zscore, x = TRUE, y = TRUE)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.063859 -0.006274  0.000033  0.007041  0.048673
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0019266  0.0004614   4.175 3.41e-05 ***
## SPY          0.0224507  0.0036400   6.168 1.27e-09 ***
## MDY         -0.0100001  0.0013136  -7.613 1.03e-13 ***
## QQQ         -0.0082541  0.0016816  -4.908 1.18e-06 ***
## DIA          0.0090240  0.0020522   4.397 1.29e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01141 on 607 degrees of freedom
## Multiple R-squared:  0.6418, Adjusted R-squared:  0.6394
## F-statistic: 271.9 on 4 and 607 DF,  p-value: < 2.2e-16
```

Note the two regressions have identical results for t-stats, R-Square.

## 6. PCA of Explanatory Variables

### 6.1 Extract scaled x matrix (except intercept)

```
x=fit$x[,-1]
cor(x)

##           SPY           MDY           QQQ           DIA
## SPY 1.0000000 0.9175267 0.9181598 0.9540900
## MDY 0.9175267 1.0000000 0.7689490 0.9092160
## QQQ 0.9181598 0.7689490 1.0000000 0.7977508
## DIA 0.9540900 0.9092160 0.7977508 1.0000000

var(x)

##           SPY           MDY           QQQ           DIA
## SPY 1.0000000 0.9175267 0.9181598 0.9540900
## MDY 0.9175267 1.0000000 0.7689490 0.9092160
## QQQ 0.9181598 0.7689490 1.0000000 0.7977508
## DIA 0.9540900 0.9092160 0.7977508 1.0000000
```

### 6.2 Compute PCA of x

The R function `princomp()` is used below. Note that other functions performing PCA are available, e.g. `prcomp()`.

```
#library(ggplot2)
#library(ggfortify)
x.primcomp<-princomp(x)
summary(x.primcomp)

## Importance of components:
##              Comp.1      Comp.2      Comp.3      Comp.4
## Standard deviation  1.9053417 0.51045028 0.30208710 0.106400323
## Proportion of Variance 0.9090671 0.06524648 0.02285149 0.002834889
## Cumulative Proportion 0.9090671 0.97431362 0.99716511 1.000000000
```

Note available output from `princomp()` is a named list:

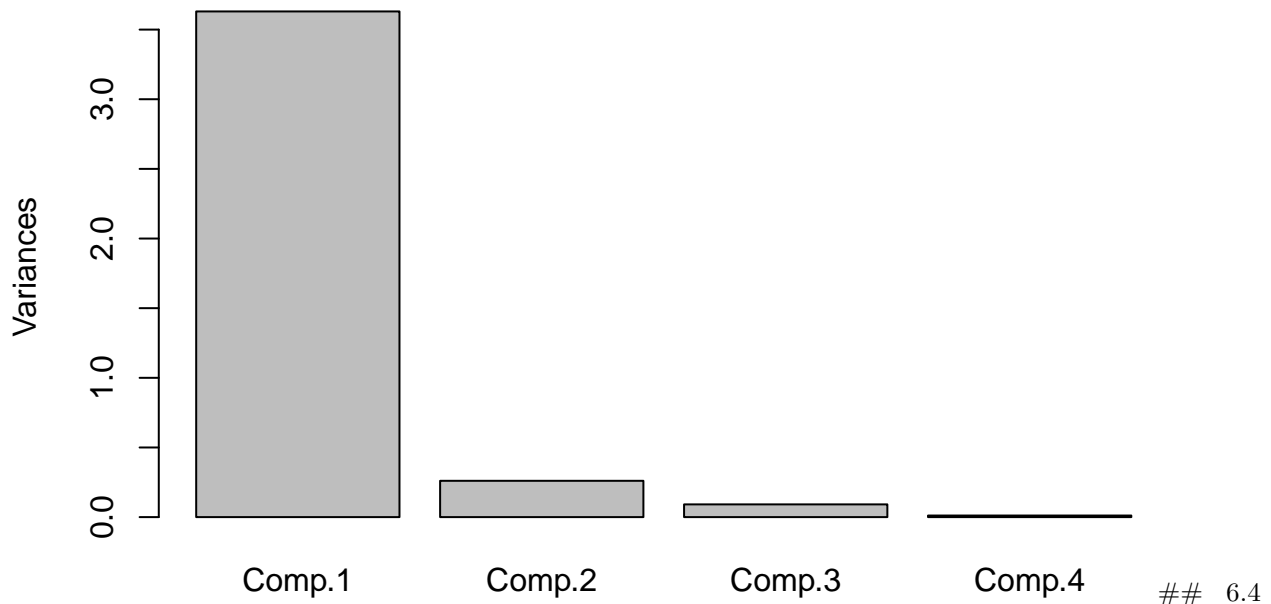
```
names(x.primcomp)

## [1] "sdev"      "loadings" "center"   "scale"    "n.obs"    "scores"   "call"
```

### 6.3 The Screeplot: Barplot of Principal Component Variances

```
screeplot(x.primcomp)
```

## x.princomp



Loadings of Principal Components

Printing out the loadings matrix (all values and with cutoff):

```
# Print out loadings
```

```
x.princomp$loadings
```

```
##
```

```
## Loadings:
```

```
##      Comp.1 Comp.2 Comp.3 Comp.4
```

```
## SPY  0.521      0.145  0.837
```

```
## MDY  0.495 -0.497 -0.699 -0.140
```

```
## QQQ  0.479  0.792 -0.155 -0.346
```

```
## DIA  0.504 -0.346  0.683 -0.400
```

```
##
```

```
##              Comp.1 Comp.2 Comp.3 Comp.4
```

```
## SS loadings      1.00  1.00  1.00  1.00
```

```
## Proportion Var   0.25  0.25  0.25  0.25
```

```
## Cumulative Var   0.25  0.50  0.75  1.00
```

```
# Get all values with no cutoffs
```

```
print(x.princomp$loadings,cutoff=0.)
```

```
##
```

```
## Loadings:
```

```
##      Comp.1 Comp.2 Comp.3 Comp.4
```

```
## SPY  0.521  0.079  0.145  0.837
```

```
## MDY  0.495 -0.497 -0.699 -0.140
```

```
## QQQ  0.479  0.792 -0.155 -0.346
```

```
## DIA  0.504 -0.346  0.683 -0.400
```

```
##
```

```
##              Comp.1 Comp.2 Comp.3 Comp.4
```

```
## SS loadings      1.00  1.00  1.00  1.00
```

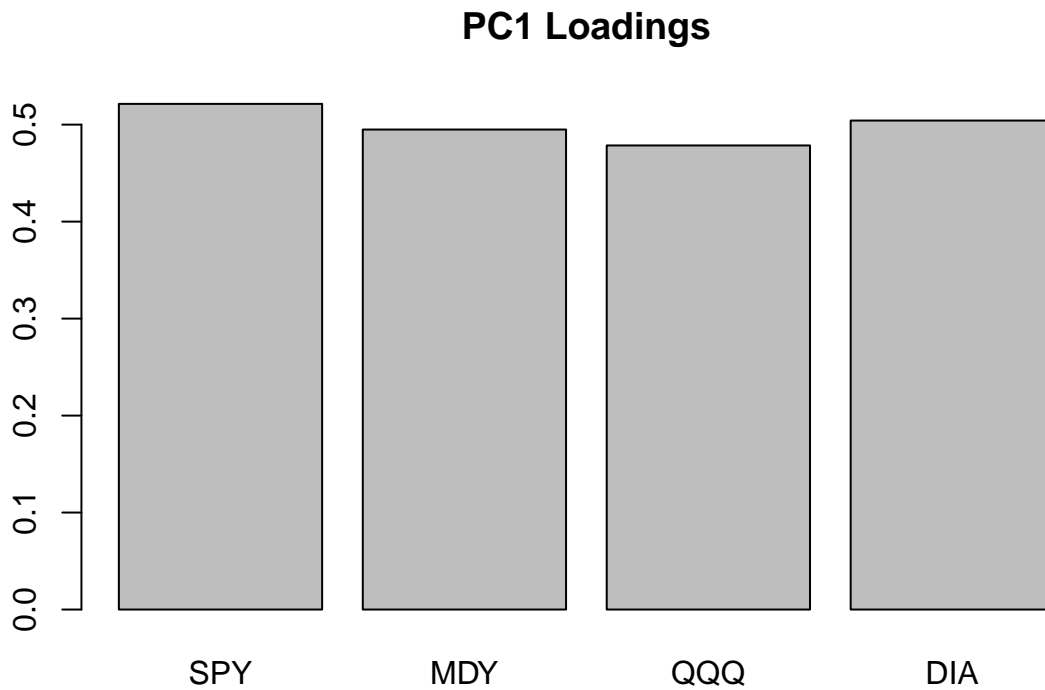
```
## Proportion Var   0.25  0.25  0.25  0.25
```

```
## Cumulative Var    0.25    0.50    0.75    1.00
```

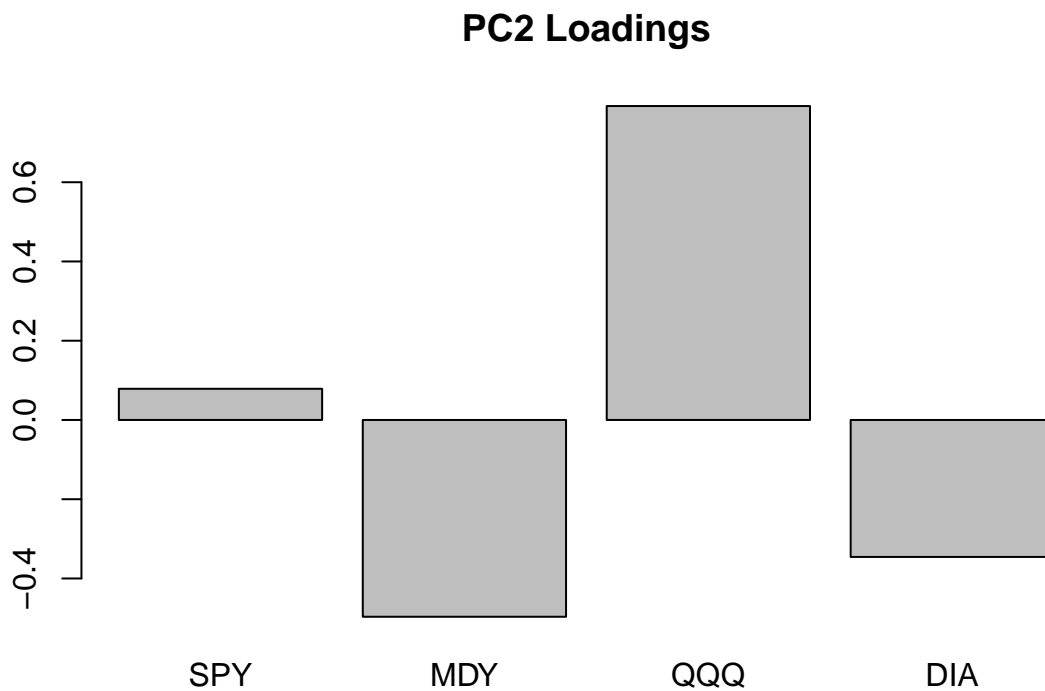
Plotting the loadings of each Principal Component variable

```
# Plot loadings of PC1-PC4
```

```
barplot(x.princomp$loadings[,1], main="PC1 Loadings")
```



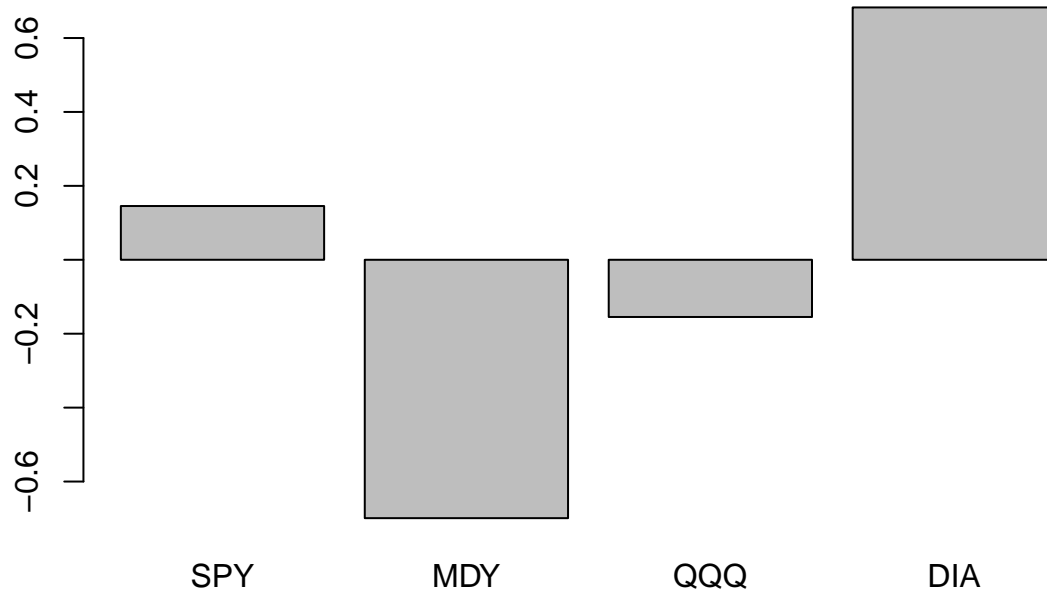
```
barplot(x.princomp$loadings[,2], main="PC2 Loadings")
```



```
barplot(x.princomp$loadings[,3], main="PC3 Loadings")
```

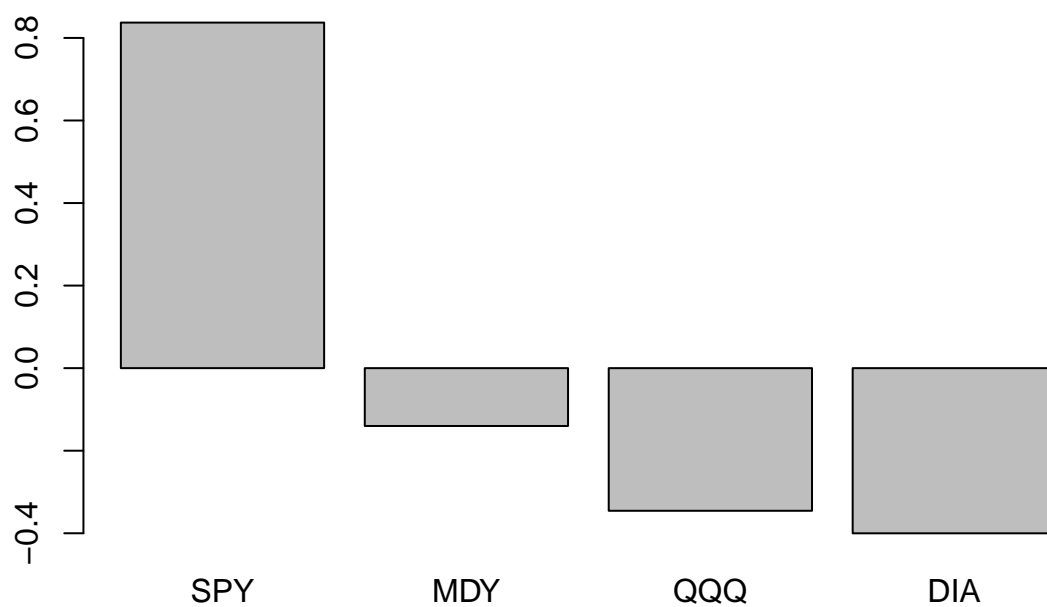


### PC3 Loadings



```
barplot(x.princomp$loadings[,4], main="PC4 Loadings")
```

### PC4 Loadings



## 7. Regressions on PC variables

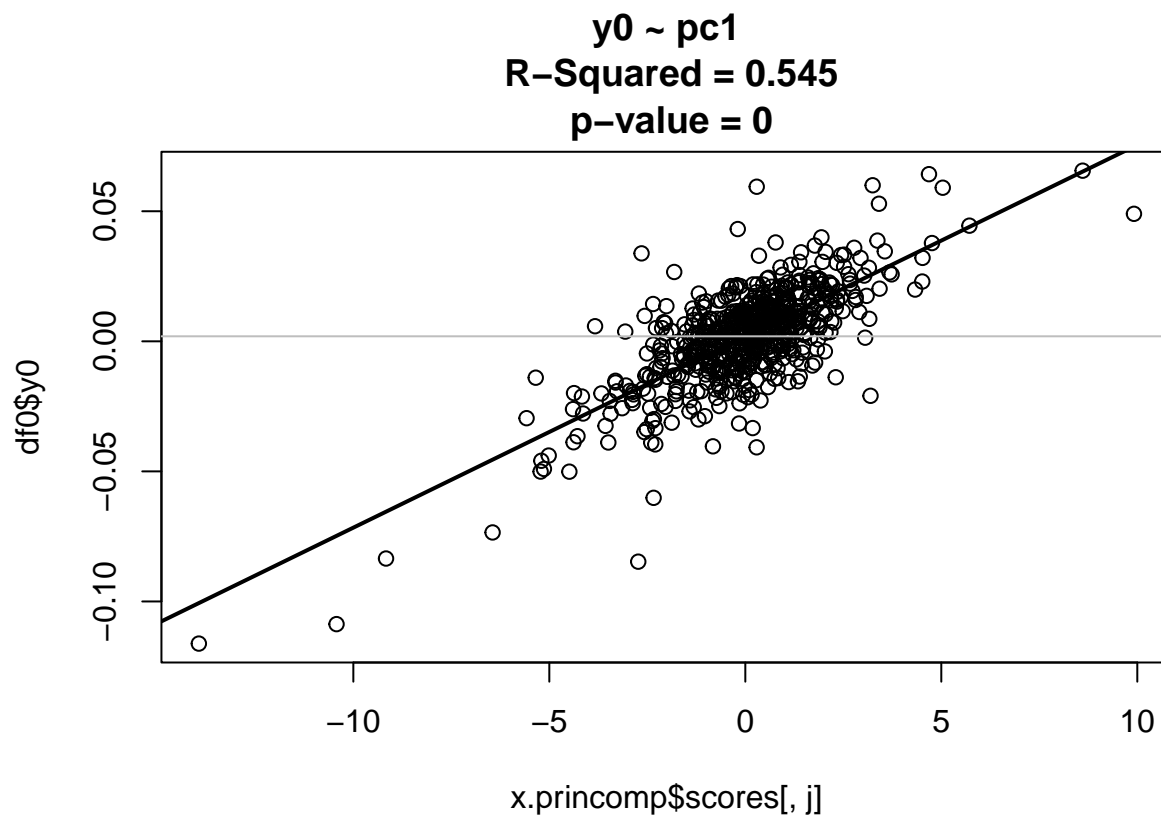
### 7.1 Univariate regressions

```
# Loop over all PC Variables
for (j in 1:NCOL(x.princomp$scores)){
  lm.j<-lm(df0$y0 ~ x.princomp$scores[,j])
  lm.j.summary<-summary(lm.j)

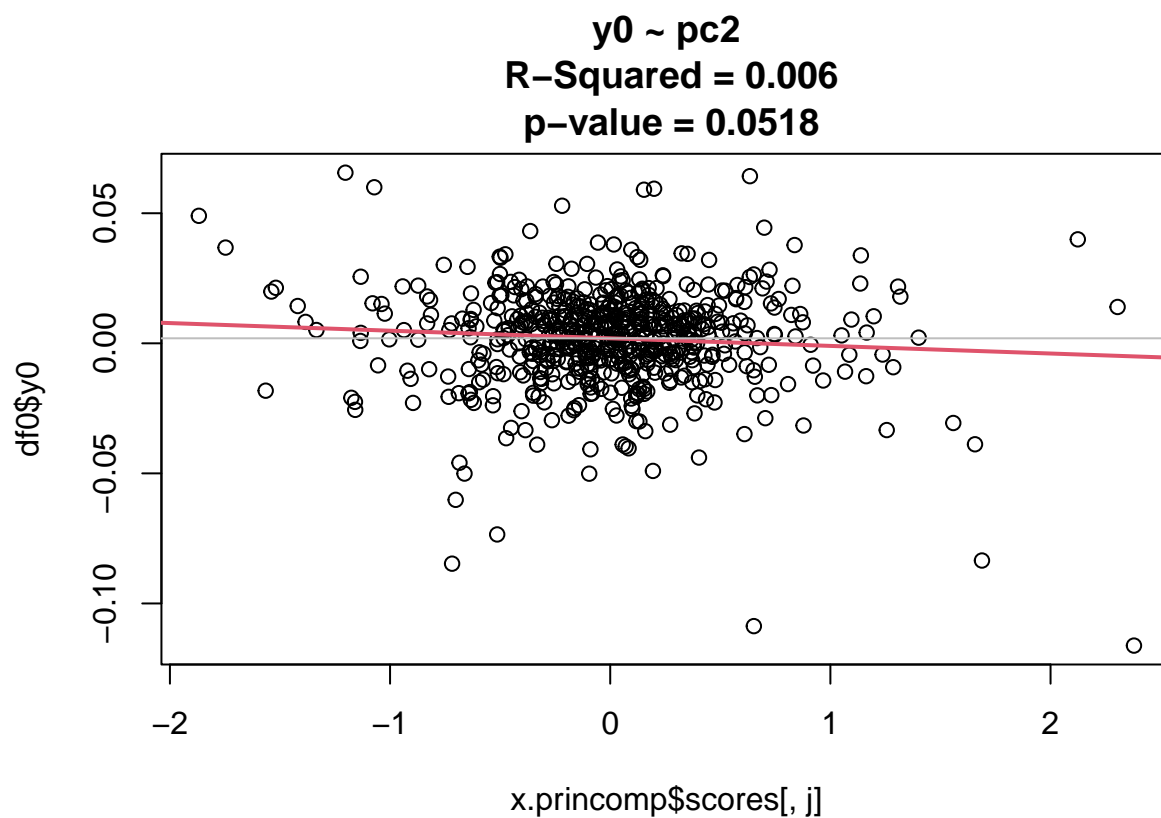
  # Use lsfit() to compute p-value s
  lsfit.j<-lsfit(x=x.princomp$scores[,j], y=y0)
  lsfit.j.print<-ls.print(lsfit.j)

  plot(x.princomp$scores[,j], df0$y0,main=c(" \n \n "))
  abline(lm.j, col=j,lwd=2)
  abline(h=mean(df0$y0),col='gray')
  title(main=paste(c("y0 ~ pc",as.character(j),
    "\n R-Squared = ", as.character(round(lm.j.summary$r.squared,digits=3)),
    "\n p-value = ", as.character(lsfit.j.print$summary[1,6])),
    collapse=""))
}

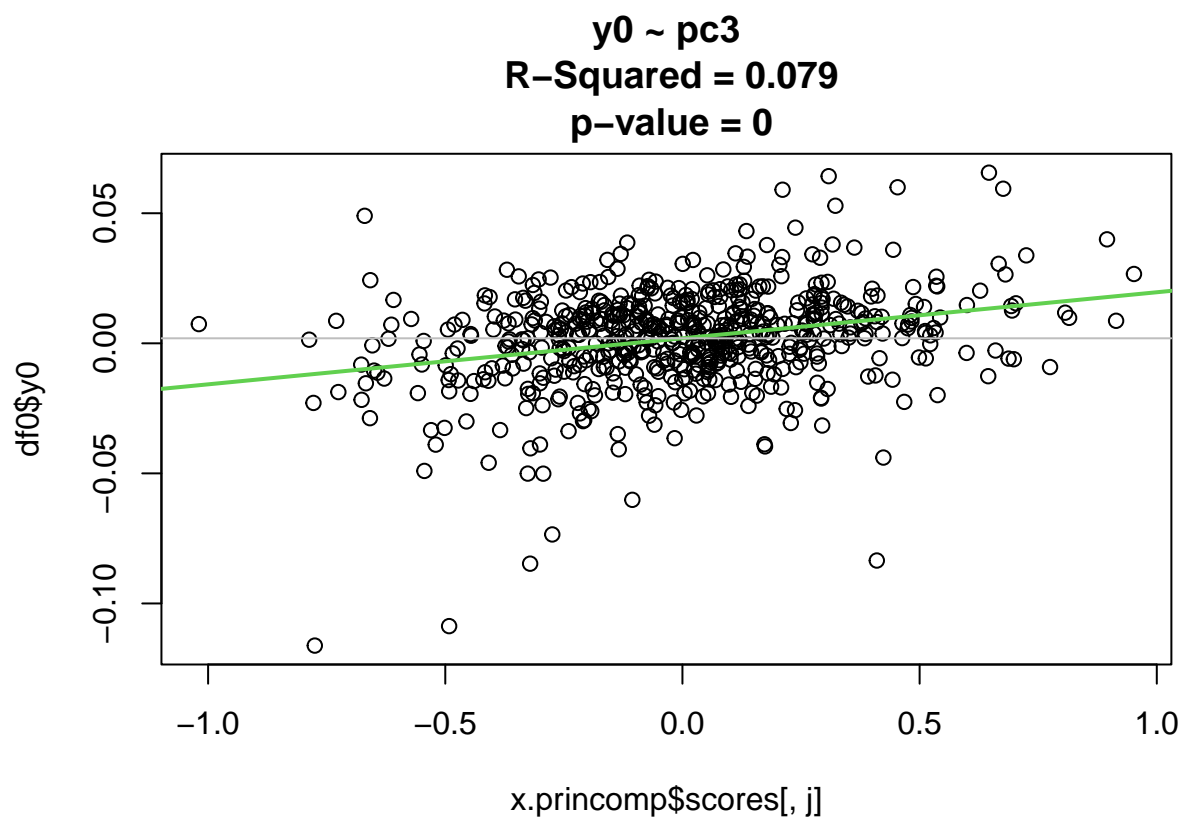
## Residual Standard Error=0.0128
## R-Square=0.5445
## F-statistic (df=1, 610)=729.3205
## p-value=0
##
##           Estimate Std.Err t-value Pr(>|t|)
## Intercept    0.0019   5e-04   3.7121    2e-04
## X             0.0074   3e-04  27.0059    0e+00
```



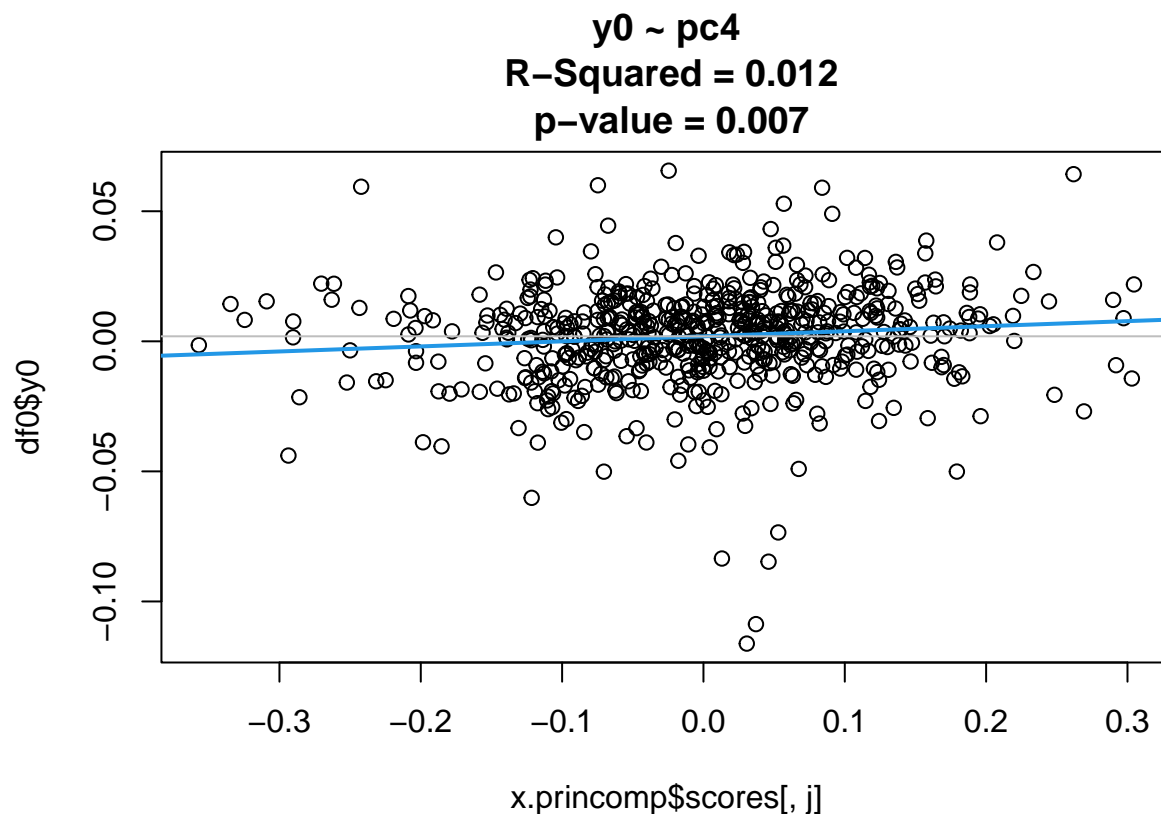
```
## Residual Standard Error=0.019
## R-Square=0.0062
## F-statistic (df=1, 610)=3.797
## p-value=0.0518
##
##           Estimate Std.Err t-value Pr(>|t|)
## Intercept  0.0019  0.0008  2.5130  0.0122
## X          -0.0029  0.0015 -1.9486  0.0518
```



```
## Residual Standard Error=0.0183
## R-Square=0.0792
## F-statistic (df=1, 610)=52.4679
## p-value=0
##
##           Estimate Std.Err t-value Pr(>|t|)
## Intercept  0.0019  0.0007  2.6107  0.0093
## X          0.0177  0.0024  7.2435  0.0000
```



```
## Residual Standard Error=0.0189
## R-Square=0.0119
## F-statistic (df=1, 610)=7.319
## p-value=0.007
##
##           Estimate Std.Err t-value Pr(>|t|)
## Intercept  0.0019  0.0008  2.5202   0.012
## X          0.0194  0.0072  2.7054   0.007
```



## 7.2 Regression model on all PC variables

```
pcregfit<-lm(df0$y0 ~ x.princomp$scores)
summary(pcregfit)
```

```
##
## Call:
## lm(formula = df0$y0 ~ x.princomp$scores)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-0.063859	-0.006274	0.000033	0.007041	0.048673

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.0019266	0.0004614	4.175	3.41e-05 ***
x.princomp\$scoresComp.1	0.0073563	0.0002422	30.377	< 2e-16 ***
x.princomp\$scoresComp.2	-0.0029266	0.0009039	-3.238	0.00127 **
x.princomp\$scoresComp.3	0.0176948	0.0015274	11.585	< 2e-16 ***
x.princomp\$scoresComp.4	0.0194375	0.0043366	4.482	8.83e-06 ***

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01141 on 607 degrees of freedom
## Multiple R-squared:  0.6418, Adjusted R-squared:  0.6394
## F-statistic: 271.9 on 4 and 607 DF, p-value: < 2.2e-16
```

```
pcregfit.summary<-summary(pcregfit)
```

### 7.3 Use PCA regression to recompute regression parameters

```
#      on original variables
pcbetavec<-as.numeric(pcregfit$coefficients[-1])
x.princomp$loadings

##
## Loadings:
##      Comp.1 Comp.2 Comp.3 Comp.4
## SPY  0.521      0.145  0.837
## MDY  0.495 -0.497 -0.699 -0.140
## QQQ  0.479  0.792 -0.155 -0.346
## DIA  0.504 -0.346  0.683 -0.400
##
##              Comp.1 Comp.2 Comp.3 Comp.4
## SS loadings      1.00  1.00  1.00  1.00
## Proportion Var   0.25  0.25  0.25  0.25
## Cumulative Var   0.25  0.50  0.75  1.00

betaFromPCA<-x.princomp$loadings %*% as.matrix(pcbetavec)
fit$coefficients

## (Intercept)      SPY      MDY      QQQ      DIA
## 0.001926583 0.022450726 -0.010000091 -0.008254136 0.009023990
```

### 7.4 Compute regression parameter based on only first 3 pc vars

```
pcbetavec

## [1] 0.007356254 -0.002926641 0.017694763 0.019437483

pcbetavec123<-0*pcbetavec
pcbetavec123[1:3]<-pcbetavec[1:3]

betaFromPC123<-x.princomp$loadings %*% as.matrix(pcbetavec123)
```

### 7.5 Compute regression parameter based on significant pc vars

The code below computes the regression parameter (original scale) using only the statistically significant pc variables, as determined by those that have  $abs\ t\ stat > \sqrt{2}$ .

```
ind.tokeep<-as.numeric(
  abs(pcregfit.summary$coefficients[-1,3])>sqrt(2))

pcbetavectokeep<-pcbetavec*ind.tokeep

betaFromPCtokeep<-x.princomp$loadings %*% as.matrix(pcbetavectokeep)
```

### 7.6 Create table of betas from these three fits

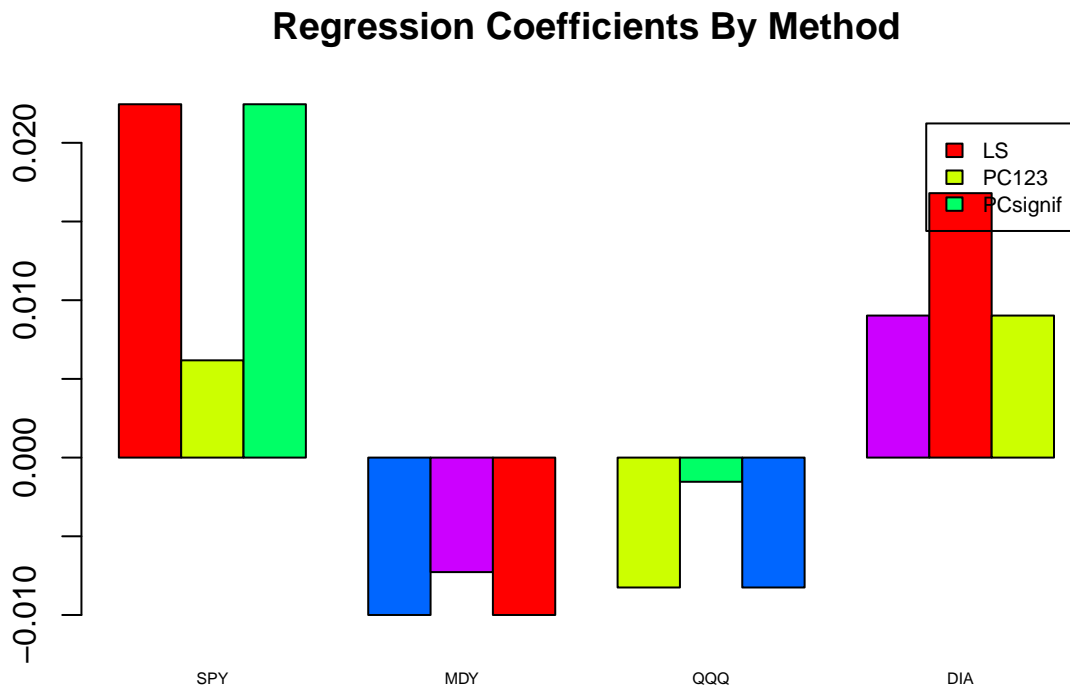
```
tab.betas<-cbind(betaFromPCA, betaFromPC123, betaFromPCtokeep)
dimnames(tab.betas)[[2]]<-c("LS", "PC123", "PCsignif")
```

```
print(tab.betas)
```

```
##           LS           PC123      PCsignif
## SPY  0.022450726  0.006178644  0.022450726
## MDY -0.010000091 -0.007275983 -0.010000091
## QQQ -0.008254136 -0.001537180 -0.008254136
## DIA  0.009023990  0.016801960  0.009023990
```

```
# 5. Compare coefficients from fits including LASSO and Ridge
```

```
barplot(t(as.matrix(tab.betas)), beside=TRUE,
        col=rainbow(5), cex.names=.5,
        legend=TRUE, args.legend=list(cex=.7))
title(main="Regression Coefficients By Method")
```

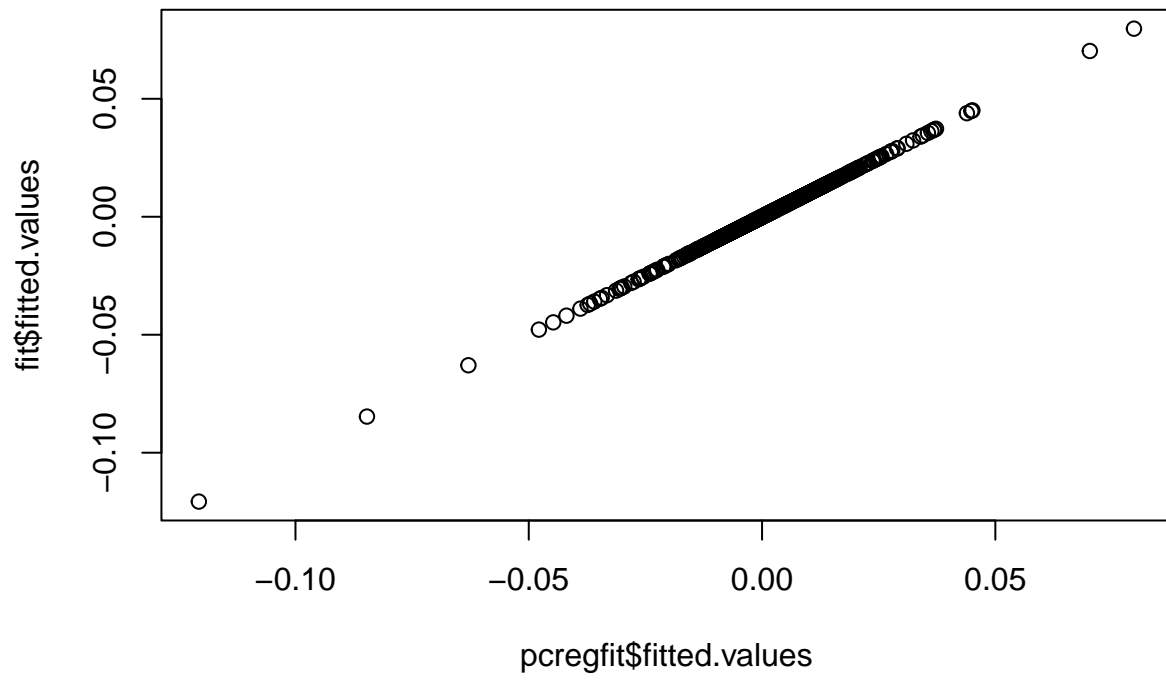


## 7.7 Demonstrate equality of LS and PCA Regression Fitted Values

```
# pcregfit equals LS fit
```

```
par(mfcol=c(1,1))
plot(pcregfit$fitted.values, fit$fitted.values)
```





## 8. Ridge and Lasso Regression Fits

### 8.1 Load glmnet package for ridge/lasso regressions

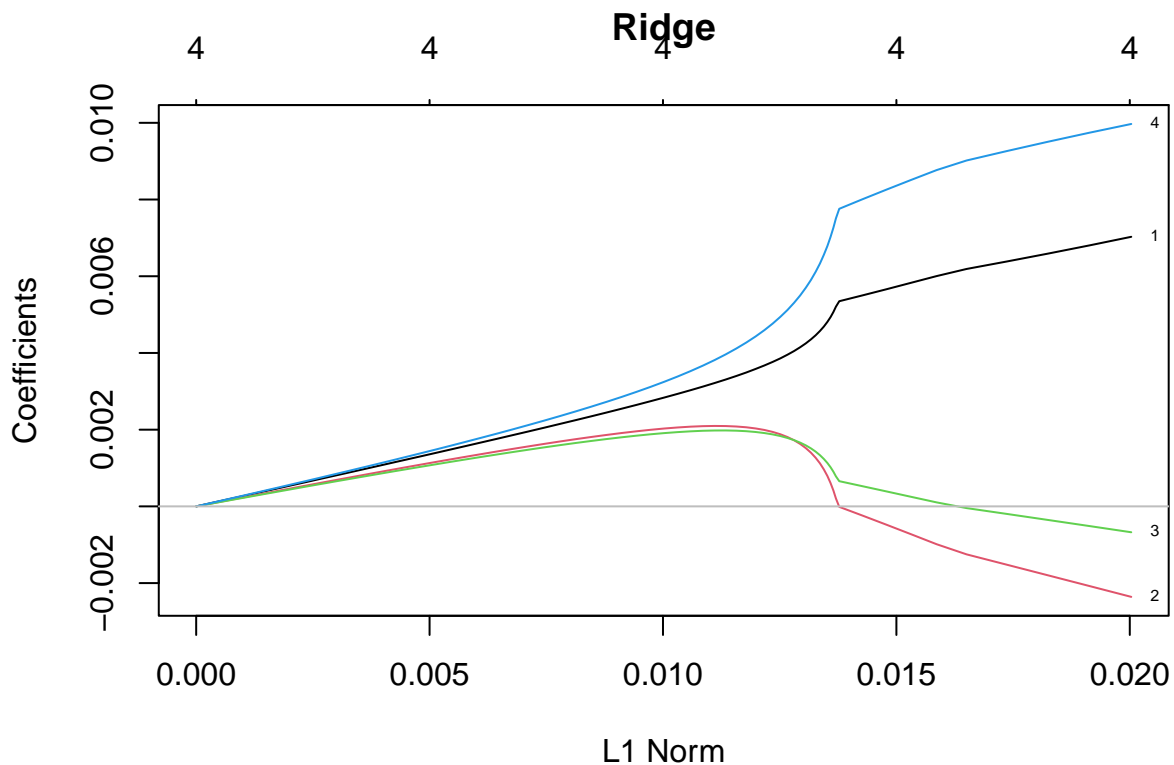
```
library(glmnet)
```

### 8.2 Define y vector and x matrix for ridge/lasso fits

```
y=fit$y  
x=fit$x[, -1]
```

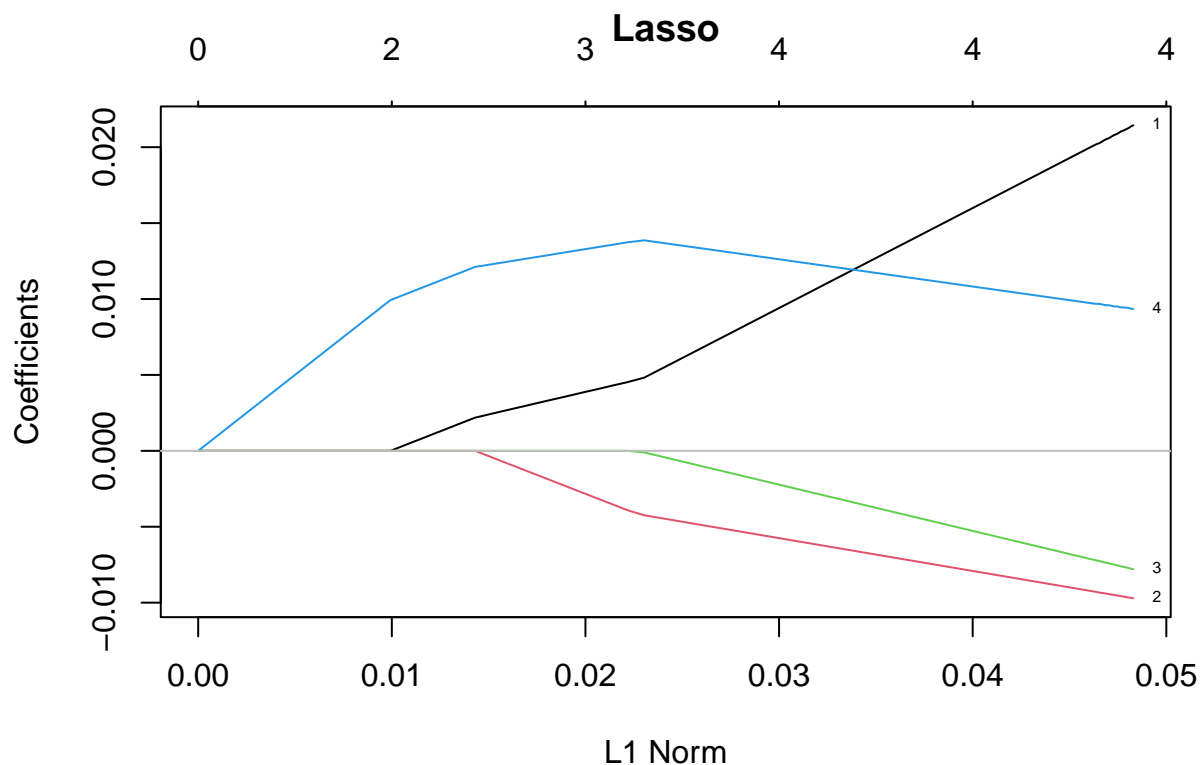
### 8.3 Plot coefficient trace of ridge regression model

```
y.glmnet.ridge<-glmnet(x,y, alpha=0)  
# alpha=0 for Ridge  
plot(y.glmnet.ridge, label=TRUE, main="Ridge")  
abline(h=0,col='gray')
```



### 8.4 Plot coefficient trace of lasso regression model —

```
y.glmnet.lasso<-glmnet(x,y, alpha=1)  
# alpha=1 for LASSO  
plot(y.glmnet.lasso, label=TRUE,main="Lasso")  
abline(h=0,col='gray')
```



## 8.5 Apply Cross Validation to choose ridge parameters

```
lambdas=10^seq(3.,-5, by=-.1)
summary(y.glmnet.ridge)
```

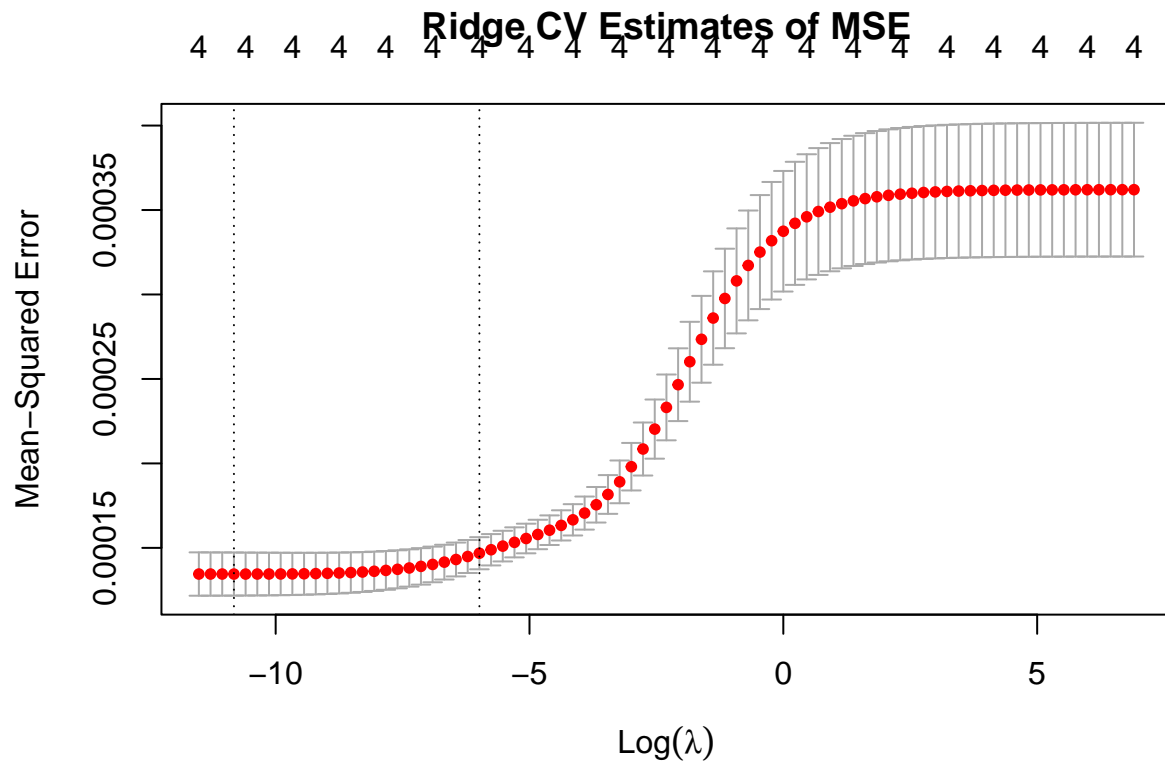
```
##          Length Class      Mode
## a0         100   -none-    numeric
## beta        400 dgCMatrix S4
## df          100   -none-    numeric
## dim           2   -none-    numeric
## lambda       100   -none-    numeric
## dev.ratio   100   -none-    numeric
## nulldev       1   -none-    numeric
## npasses       1   -none-    numeric
## jerr          1   -none-    numeric
## offset        1   -none-    logical
## call          4   -none-    call
## nobs          1   -none-    numeric
```

```
#
# Cross-validation estimates of prediction error
# ridge case (alpha=0)
y.glmnet.ridge<-glmnet(x,y,alpha=0, lambda=lambdas)
summary(y.glmnet.ridge)
```

```
##          Length Class      Mode
## a0          81   -none-    numeric
## beta        324 dgCMatrix S4
## df          81   -none-    numeric
## dim           2   -none-    numeric
```

```
## lambda      81    -none-    numeric
## dev.ratio   81    -none-    numeric
## nulldev     1    -none-    numeric
## npasses     1    -none-    numeric
## jerr        1    -none-    numeric
## offset      1    -none-    logical
## call        5    -none-    call
## nobs        1    -none-    numeric
```

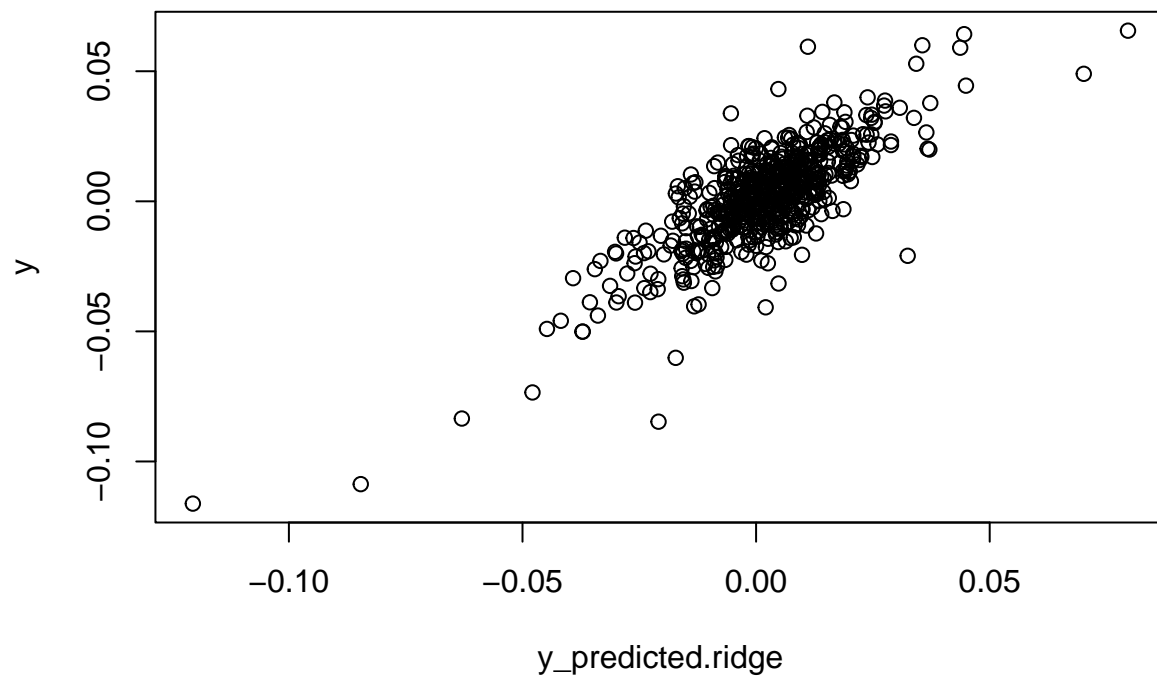
```
y.cv.glmnet.ridge<-cv.glmnet(x,y,alpha=0, lambda=lambdas)
plot(y.cv.glmnet.ridge, main="Ridge CV Estimates of MSE")
```



```
optlambda.ridge<-y.cv.glmnet.ridge$lambda.min
glmnet.ridgefit<-y.cv.glmnet.ridge$glmnet.fit
y_predicted.ridge<-predict(glmnet.ridgefit,s=optlambda.ridge,newx=x)
```

Plot Observed vs Fitted for Ridge Regression

```
plot(y_predicted.ridge, y)
```

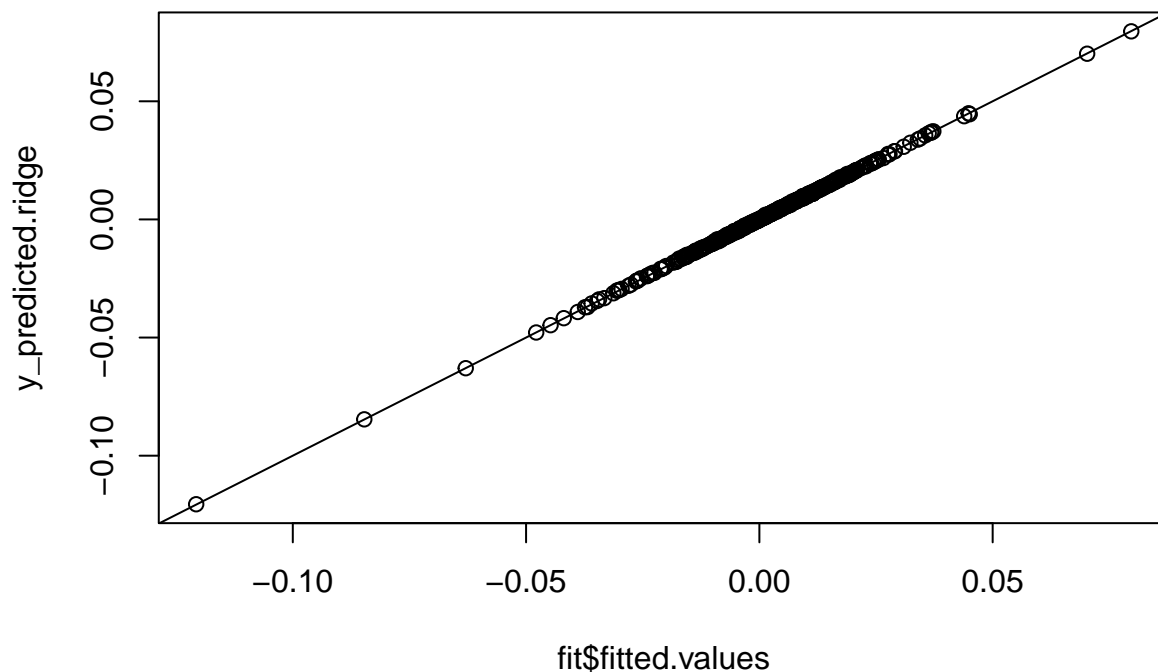


```
cor(y_predicted.ridge,y)^2
```

```
##           [,1]
## s1 0.6416741
```

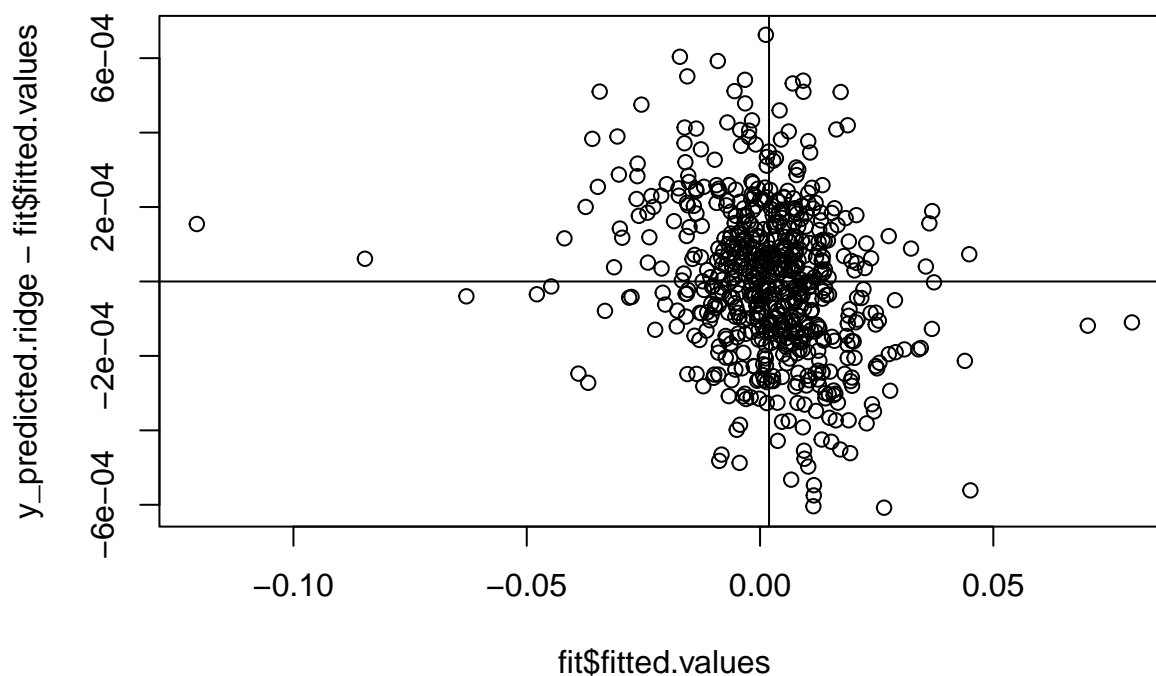
```
# Compare ridge fitted to ls fitted
plot(fit$fitted.values, y_predicted.ridge,
     main="Ridge Regression\n(Shrinks to Mean)")
abline(a=0,b=1)
```

### Ridge Regression (Shrinks to Mean)



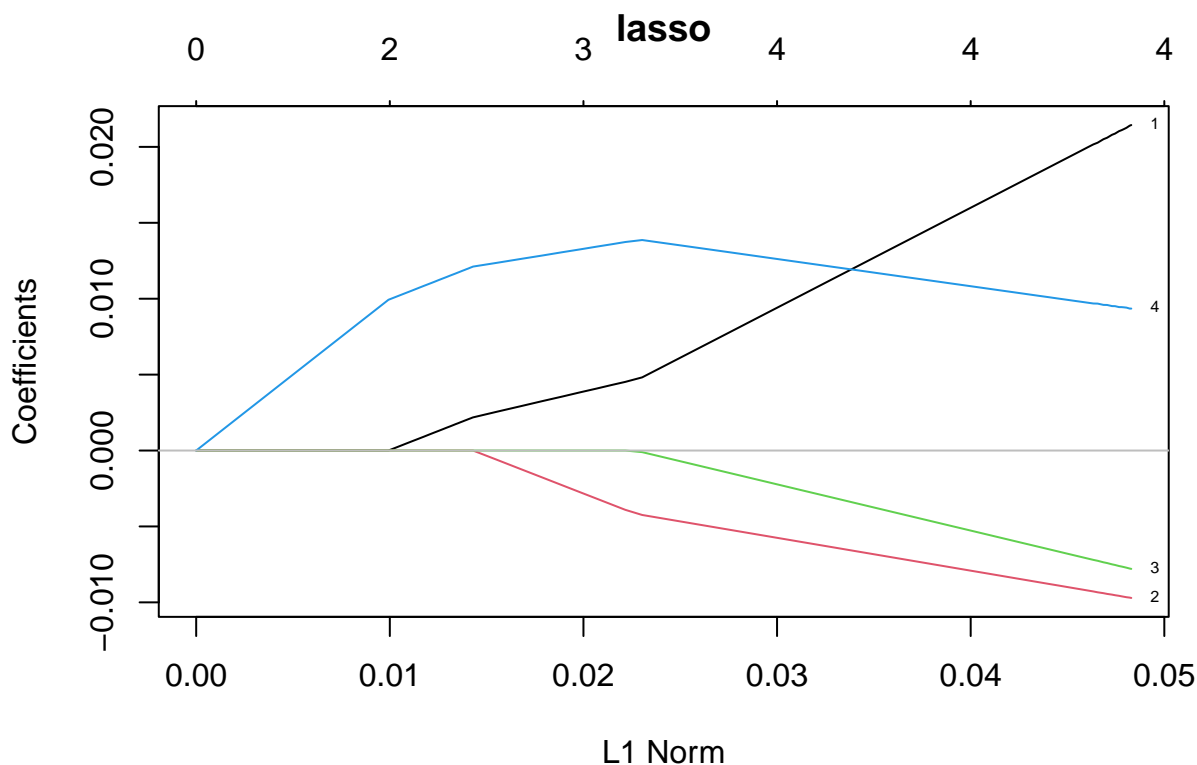
```
plot(fit$fitted.values, y_predicted.ridge ~ fit$fitted.values,
     main="Ridge Regression \n(Shrinks to Mean)")
abline(h=0)
abline(v=mean(fit$fitted.values))
```

### Ridge Regression (Shrinks to Mean)



## 8.6 Apply Cross Validation to choose lasso parameters

```
y.glmnet.lasso<-glmnet(x,y, alpha=1)
# alpha=1 for lasso
plot(y.glmnet.lasso, label=TRUE, main="lasso")
abline(h=0,col='gray')
```



```
lambdas=10^seq(3.,-5, by=-.1)
summary(y.glmnet.lasso)
```

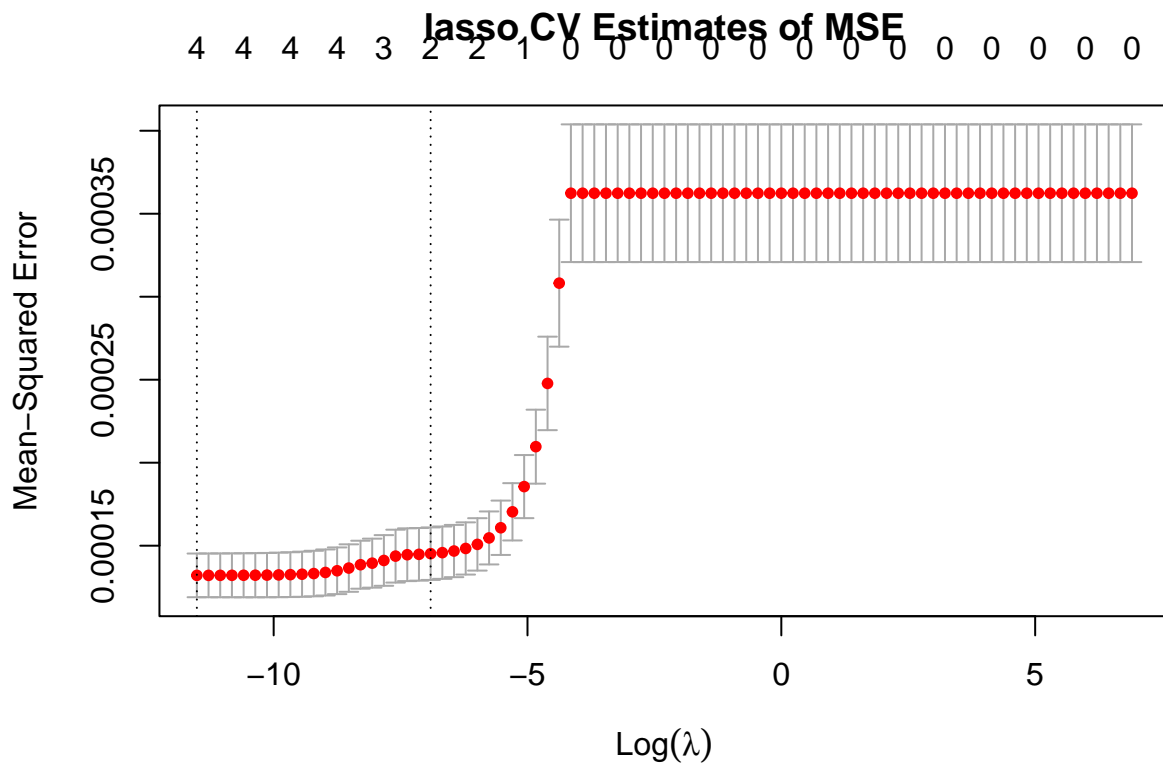
```
##          Length Class      Mode
## a0         79    -none-  numeric
## beta       316 dgCMatrix S4
## df         79    -none-  numeric
## dim         2    -none-  numeric
## lambda      79    -none-  numeric
## dev.ratio   79    -none-  numeric
## nulldev     1    -none-  numeric
## npasses     1    -none-  numeric
## jerr        1    -none-  numeric
## offset      1    -none-  logical
## call        4    -none-  call
## nobs        1    -none-  numeric
```

```
#
# Cross-validation estimates of prediction error
# lasso case (alpha=1)
y.glmnet.lasso<-glmnet(x,y,alpha=1, lambda=lambdas)
```

```
summary(y.glmnet.lasso)
```

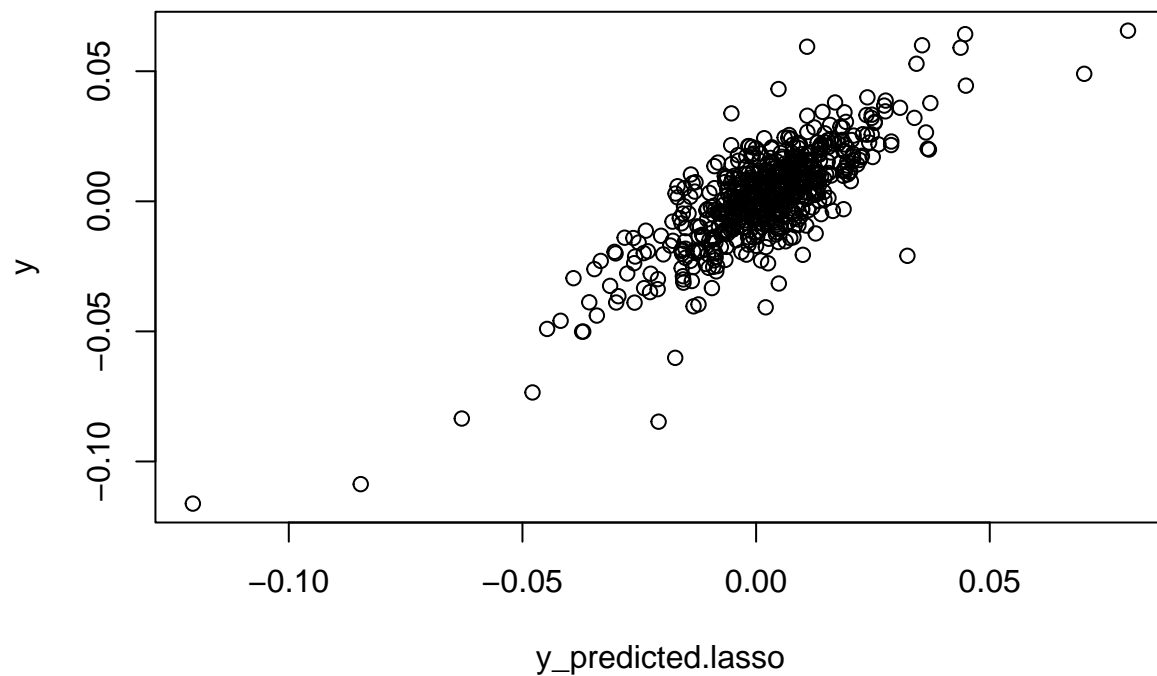
```
##          Length Class      Mode
## a0         81    -none-   numeric
## beta      324 dgCMatrix S4
## df         81    -none-   numeric
## dim         2    -none-   numeric
## lambda     81    -none-   numeric
## dev.ratio  81    -none-   numeric
## nulldev     1    -none-   numeric
## npasses     1    -none-   numeric
## jerr        1    -none-   numeric
## offset      1    -none-   logical
## call        5    -none-   call
## nobs        1    -none-   numeric
```

```
y.cv.glmnet.lasso<-cv.glmnet(x,y,alpha=1, lambda=lambdas)
plot(y.cv.glmnet.lasso, main="lasso CV Estimates of MSE")
```



```
optlambda.lasso<-y.cv.glmnet.lasso$lambda.min
glmnet.lassofit<-y.cv.glmnet.lasso$glmnet.fit
y_predicted.lasso<- predict(glmnet.lassofit,s=optlambda.lasso,newx=x)
#
plot(y_predicted.lasso, y)
```



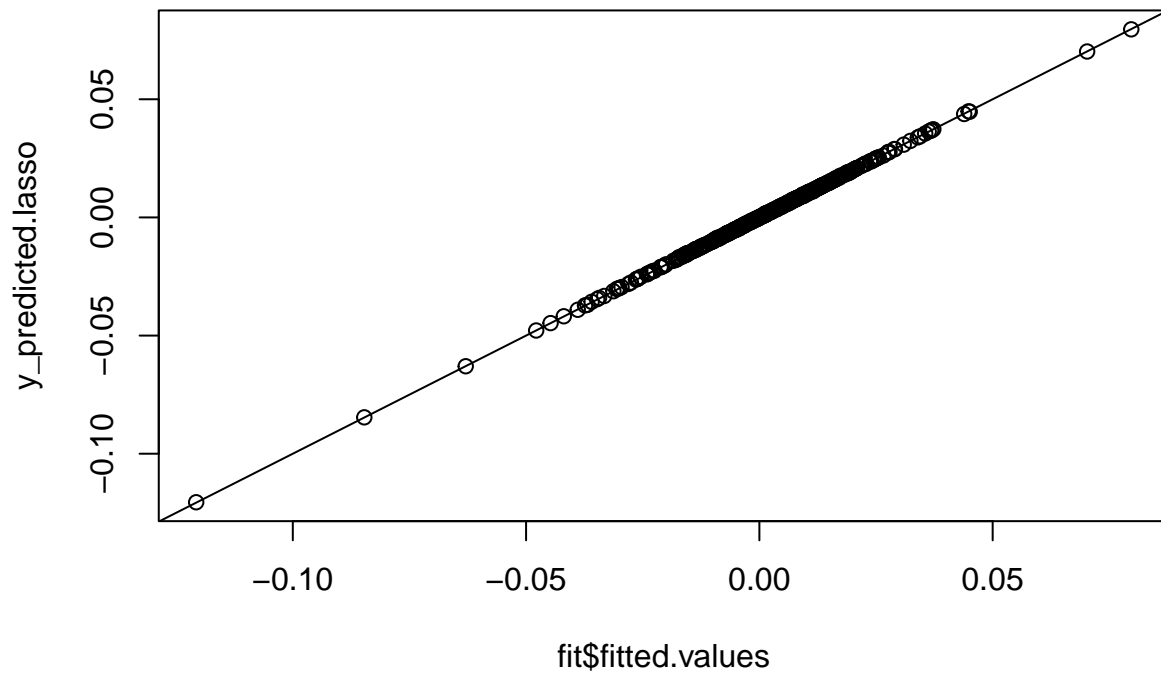


```
cor(y_predicted.lasso,y)^2
```

```
##           [,1]  
## s1 0.6417391
```

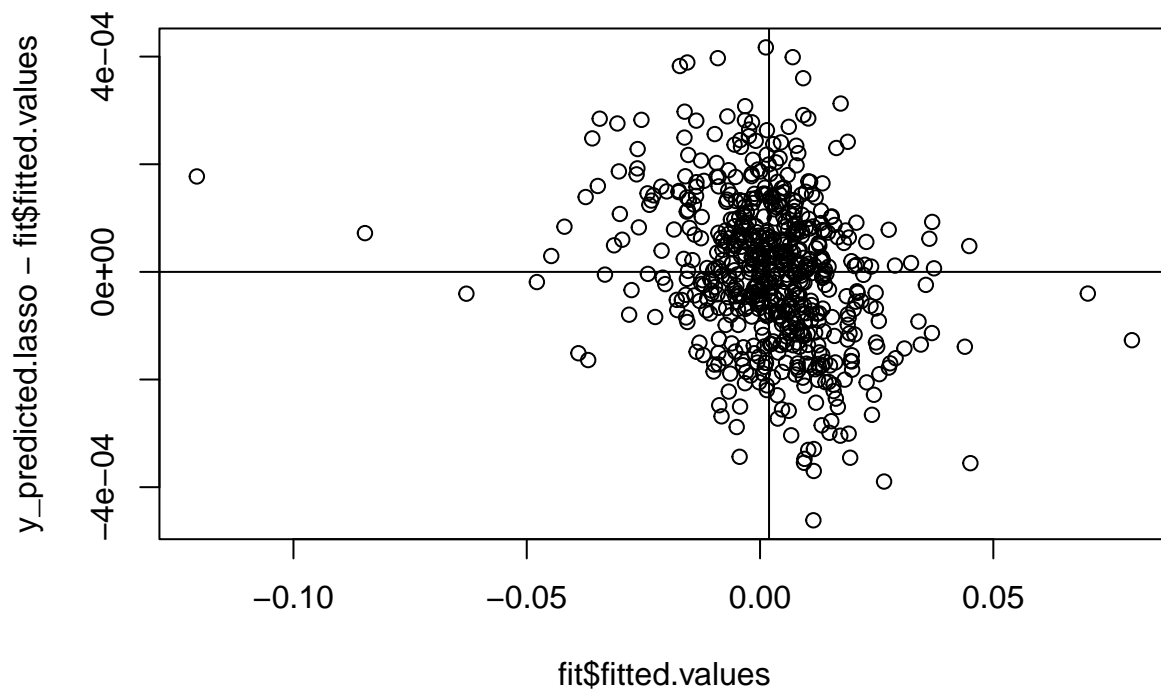
```
# Compare lasso fitted to ls fitted  
plot(fit$fitted.values, y_predicted.lasso,  
     main="lasso Regression \n(Shrinks to Mean)")  
abline(a=0,b=1)
```

### lasso Regression (Shrinks to Mean)



```
plot(fit$fitted.values, y_predicted.lasso ~ fit$fitted.values,  
     main="lasso Regression \n(Shrinks to Mean)",  
     abline(h=0, v=mean(fit$fitted.values)))
```

### lasso Regression (Shrinks to Mean)



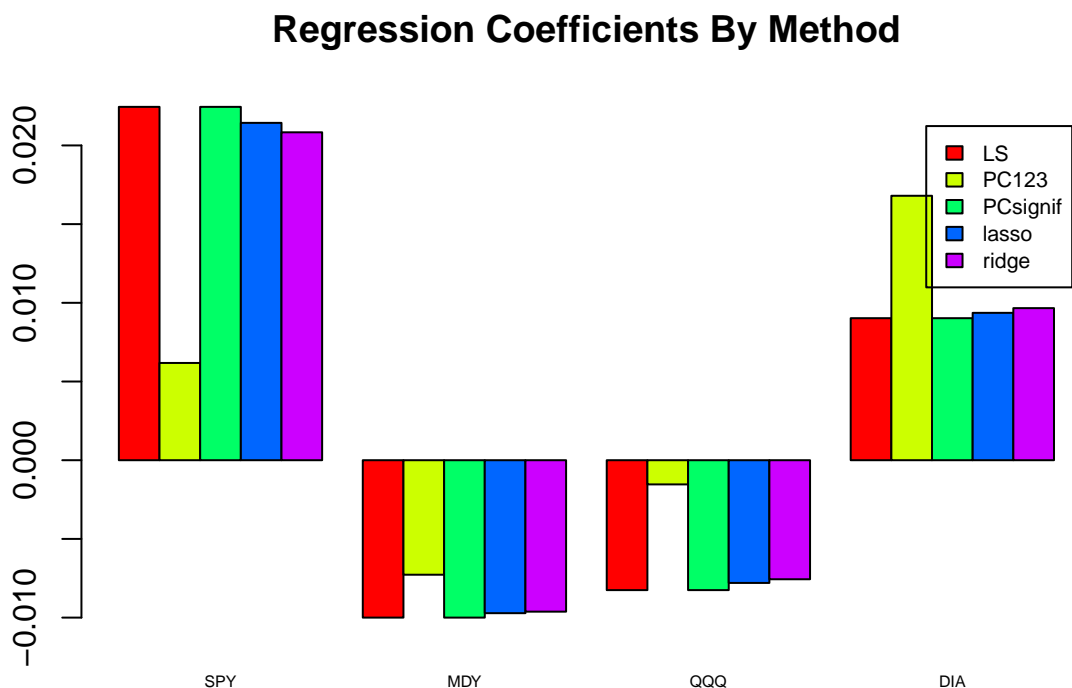
```
## 8.7 Compare coefficients from fits including LASSO and Ridge ----
```

```
tab.betas<-cbind(betaFromPCA, betaFromPC123, betaFromPCtokeep)
dimnames(tab.betas)[[2]]<-c("LS", "PC123", "PCsignif")
print(tab.betas)
```

```
##           LS           PC123           PCsignif
## SPY  0.022450726  0.006178644  0.022450726
## MDY -0.010000091 -0.007275983 -0.010000091
## QQQ -0.008254136 -0.001537180 -0.008254136
## DIA  0.009023990  0.016801960  0.009023990
```

```
tab.betas2<-data.frame(cbind(tab.betas,
                             lasso=coef(y.cv.glmnet.lasso,s="lambda.min")[-1],
                             ridge=coef(y.cv.glmnet.ridge,s="lambda.min")[-1]))
```

```
barplot(t(as.matrix(tab.betas2)), beside=TRUE,
        col=rainbow(5), cex.names=.5,
        legend=TRUE, args.legend=list(cex=.7))
title(main="Regression Coefficients By Method")
```



MIT OpenCourseWare  
<https://ocw.mit.edu>

18.642 Topics in Mathematics with Applications in Finance  
Fall 2024

For information about citing these materials or our Terms of Use, visit: <https://ocw.mit.edu/terms>.